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STUDENT VERSION
COLD PILL

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Abstract: A model for the flow of a cold pill drug through the gastrointestinal compartment to the bloodstream compartment of a human subject is proposed. Students solve the system of differential equation model, use known parameter values, and plot solutions. Students also determine maximum amount of drug in the bloodstream and the time of such a peak level. Finally, students determine the rate at which the drug is absorbed into the bloodstream from the gastroinstestinal compartment in order to meet certain medical requirements, namely, set time for maximum amount of the drug in the bloodstream.

SCENARIO DESCRIPTION

This modeling opportunity is based on an examination question, offered in quotes, in a differential equations course given at the University of Delhi. We are grateful to our colleague DR Swarn Singh, Mathematics, Sri Venkateswara College, University of Delhi, Delhi INDIA for sharing this material.

"Consider a model to describe the levels of the drug in a patient taking a single cold pill:

$$\begin{aligned} \frac{dx}{dt} &= -k_1 x \\ \frac{dy}{dt} &= k_1 x - k_2 y \end{aligned} \tag{1}$$

Here k_1 and k_2 describe rates at which the drugs move between the two sequential compartments (gastroinstestinal (GI)-tract and blood stream). At time t, $x = x(t)$ and $y = y(t)$ are the levels of the drug in Gi-tract and bloodstream, respectively.

(a) Find solution expressions for $x = x(t)$ and $y = y(t)$ which satisfy this pair of differential equations, when k_1 and k_2 are not equal."

We add the following questions:

(b) For parameter values $k_1 = 0.9$ 1/hr and $k_2 = 0.6$ 1/hr with initial amounts of the drug being $x(0) = 300$ mg and $y(0) = 0$ mg plot the solutions.

(c) For parameter values $k_1 = 0.9$ and $k_2 = 0.6$ 1/hr with initial amounts of the drug being $x(0) = 300$ mg and $y(0) = 0$ mg determine when the amount of drug peaks in the bloodstream and the amount of the drug in both the bloodstream and the GI-tract at this time.

(d) Pharmaceutical manufacturers have the capability of slowing down the rate at which the drug goes from the GI-tract into the bloodstream, i.e. altering k_1, through coating on the pill for example. This will not alter the natural rate at which the drug leaves the bloodstream or is consumed by the body. Determine how sensitive the peak amount of drug in the bloodstream is, as well as the time of that peak amount, in terms of the value of k_1. You may presume all other values are held to the same level, i.e. keep $k_2 = 0.6$ 1/hr, $x(0) = 300$ mg, and $y(0) = 0$ mg.

(e) What value of k_1, hence what coating level on the ingested pill, should the pharmaceutical company select so that the time of the maximum amount of drug in the bloodstream is 1.5 hr and the actual amount of drug in the bloodstream is 125 mg?