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STUDENT VERSION
Two Spring Mass

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Abstract: We ask students to build a Free Body Diagram for a vertical two mass situation in which the two masses are held fixed at the tip and at the bottom. The mass holds the springs together at the join of the two springs in between. The motion of the mass is tracked under differing situations, e.g., different spring parameters.

SCENARIO DESCRIPTION

We consider two springs (coils - top one thin as it has smaller spring constant and bottom one thicker as it has larger spring constant) and a mass (blue plate) in a vertical mode configuration as depicted below. The figures below in Figure 1 show on the left when the spring system is in static equilibrium, i.e. at rest, while the figure on the right has the mass displaced vertically down from static equilibrium.

Here is your set of Activities:

1. Build a mathematical model for the displacement $y(t)$ of the mass in this configuration, using Newton's Second Law with a Free Body Diagram and the named parameters above. Show that you come up with the second order differential equation (1) for $y(t)$ the displacement from static equilibrium of the mass:

$$my''(t) + (c_1 + c_2)y'(t) + (k_1 + k_2)y(t) = 0 \tag{1}$$

   where m is the mass at the point where the two masses join, k_1 is the spring constant of Spring 1, k_2 is the spring constant of Spring 2, c_1 is the coefficient of resistance constant of Spring 1, and c_2 is the coefficient of resistance constant of Spring 2,

2. Convert the second order differential equation into a system of linear first order differential equations using the 'key" $x(t) = y'(t)$.

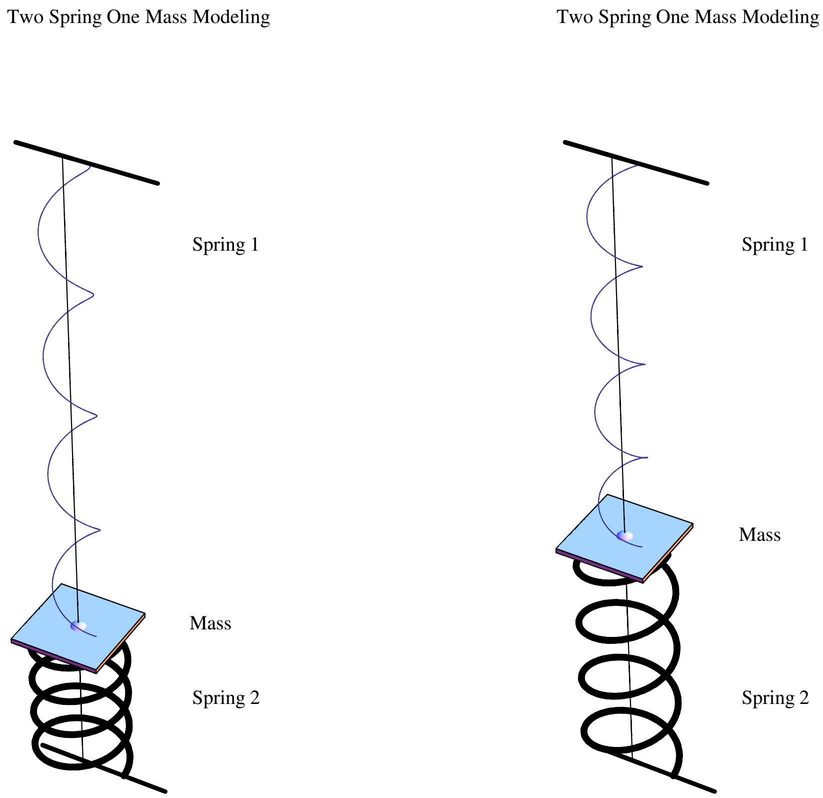

Figure 1. Two spring mass system with thin spring on top when the mass is at the static equilibrium, i.e. at rest (left) and the mass is displaced vertically down from the static equilibrium (right).

3. Use these values for the parameters in the problem, $m = 10$, $k_1 = 20$, $k_2 = 30$, $c_1 = 5$, $c_2 = 5$, $y(0) = y_0 = -18$, and $y'(0) = v_0 = 0$. Solve this system using an eigenvalue and eigenvector approach completely!
4. Solve this model for $y(t)$ from the second order differential equation you obtained in (1).
5. Using the numbers in (3) compare the eigenvalue eigenvector solution for $y(t)$ found in (3) with that found in (4).
6. Using some tool to get a plot of the displacement, $y(t)$, over the time interval $[0, 20]$ sec.
7. Here are some further questions to consider. Take on as many as assigned.
   a) What would happen if we changed both c_1 and c_2 (equally, differently)?
   b) What if both c_1 and c_2 were 0? Could they be in reality?
   c) What would happen if we changed both k_1 and k_2 (equally, differently)?
   d) What would happen if we changed y_0? Say making it positive?

e) What would happen if we changed the mass, m?

f) What would happen if we used a $y'(0)$ NOT 0?