µProject 3

Bottom Line

A mine roller has hit an IED, and I have video of how high the mine roller came off the ground. Through the USMA Math Modeling Process, I will create a mathematical model for the distance traveled by the mine roller (vertical displacement) with a particular focus put on the assumptions made to create this model, the solution techniques for the model, and the methods of approach for calculating the force of the IED's blast.

Transform

The key governing principle of this problem is that the problem is essentially a sum of forces problem. There are various forces acting on the mine roller; these include, but are not limited to, the forces of the IED blast, the weight of the mine roller, the effects of drag on the mine roller, and the nature of the attachment mechanism from the mine roller to MRAP (the strength of the metal, the force it imparts to the ground, the design of the locking pins, etc). All of these forces act in a vertical manner, with the only positive force being the IED blast (see Figure 1). Therefore: $\sum F = F_{IED} - F_G - F_D - F_A$, with each respective force previously identified. Important to note at this step, that all motion in the x direction is assumed to insignificant, and that the IED blast acts only in the vertical direction. Furthermore, this can be simplified using a combination of basic physics principles and an understanding of differential equations. Force equals mass multiplied by acceleration, and the second and first derivatives of a position function are acceleration and velocity, respectively. With this knowledge, this equation can be rewritten as $my''(t) = F_{IED} - mg - \gamma y'(t) - F_A$, where m is the mass of the mine roller, g is the magnitude of the acceleration of gravity, γ is the drag coefficient, y' is the vertical velocity, and y'' is the vertical acceleration, with all pieces of the equation in respect to t (time). In addition, m and γ are considered to be nondepreciating constants. Now, to correctly model this, two final assumptions must be made. The first being that F_{IED} is an instantaneous force that essentially imparts an initial velocity. The second being that F_A will cease to affect the mine roller once it has disconnected from the vehicle (assuming the blast is strong enough). This gives the final models in piecewise notation: $my''(t) = -mg - \gamma y'(t) - F_A$ for $t \leq T$, $my''(t) = -mg - \gamma y'(t)$ for $t \geq T$, with $y'(0) = I$, T being the time at which the attachment breaks and I being an initial velocity determined by F_{IED}.

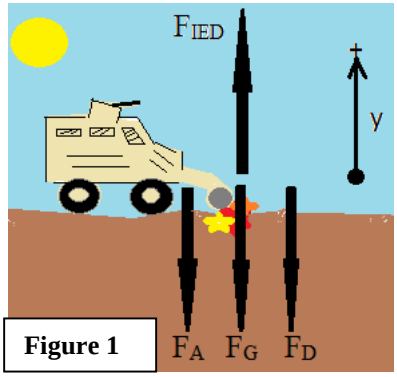

Figure 1

Solve

So, in principle, this Sum of Forces problem is actually an initial value problem of a second order, nonhomogeneous differential equation with nonconstant and constant coefficients, as at least one force, F_A, changes with respect to time, but is not present in both models. To solve this, one can use a computer algebra system, such as Mathematica, to determine the general solution for both equations, minus the force of the IED ($y'(0) = I$) and the C values.

Interpret

To solve for F_{IED}, one can take the initial value of height for the mine roller, and using the post-detach position equation in the Solve step, one can solve for t for the input maximum height attained by the mine roller where the velocity curve equals 0. With this knowledge, one can then find the initial values that the IED blast imparts on the velocity of the mine roller in an upward direction.

Improved Seating

Background

In the military, there is a vehicle called the MRAP (Mine Resistant Ambush Protected). This vehicle is used for transportation in situations where IEDs are a threat. They are equipped with seats that act as shock absorbers for the passengers when a mine does go off. Other vehicles in the military are not equipped with these seats. My job is to mathematically model the force of the blast on a passenger in the MRAP, and from this model, interpret how important these seats are to have in the all vehicles. The way the shock absorbing system works for the seats is essentially a spring with damping.

Transform

For this situation I need a mathematical model. The basic model I will use is a damped and forced equation. Since the force of the blast will be changing for the entire model (decreasing), I will call this G(t) for the time being and will be measured in Newtons. U(t) will be the displacement of the passenger from equilibrium and will be measured in meters. The damping constant will be r, and the spring coefficient will be k. The basic differential equation for a forced and damped spring is

$$mU''(t) + rU'(t) + kU(t) = G(t)$$

The assumptions for this model to work are as follows. First, we must assume that the blast generates a force that rapidly decreases as t increases, and the equation for it can be determined. A good guess would be Ae^{-t} where A is an unknown constant. Next, we must assume that mass and the spring coefficient will be given because these can easily be calculated. Finally, we need to assume that for the value r, we want a critically damped situation, and that value will also be given to us. We want it critically damped because that will dissipate the most energy the quickest, not oscillate the passenger as much as overdamped or underdamped, and return him/her to his/her initial height.

Solve

My differential equation will look like

$$mU''(t) + rU'(t) + kU(t) = Ae^{-t}$$

To solve, I would have to go about it as I would with a normal non-homogenous with constant coefficients second order differential equation. Since it is critically damped, my y_c needs to be a guess that would fit for a repeated roots characteristic equation solution. So $r^2 = 4mk$ must be a valid assumption to find r. This will allow me to solve my differential equation and determine the amount of force in the form of distance traveled by the passenger the IED will have based upon the blast.

Interpret

This differential will generate an equation that models the displacement of the passenger in respect to time. To decide on how effective these seats are, I would need to know how much passengers are displaced because of the explosion without the resistant seating. I can then compare how much of an impact these seats have on the force on the soldier. With this information I will be able to give advice to the military on whether these seats should be incorporated into all military vehicles.

µProject 3 – IEDs and Blast Resistance

Bottom Line

You are able to develop a differential equation model using Newton's Second Law, $\sum F = m \cdot a$. Your battle buddy said that he has been able to compute how high the mine roller traveled vertically after the blast from watching a nearby surveillance system. Knowing this information, you can use the internet to find the values of other parameter values such as drag (F_D) and the force of the attachment (F_a), to solve for the blast force of the IED.

Transform

This model determines the vertical displacement of a mine roller after it makes contact with an IED. The independent variable is time (t, seconds) and the dependent variable is vertical displacement (x, meters). The basic principle that governs this scenario is based off of Newton's Second Law, $\sum F = m \cdot a$. In other words, you are using a model that solves for the vertical displacement. From the work of your battle buddy, you know the vertical displacement. If you find the remaining parameter values in your solution, the only term unsolved for will be the blast force of the IED (F_{IED}). The assumptions of this model are: the mine roller only travels in the vertical direction after the IED explodes, the attachment of the mine roller does not act as a fulcrum, the mine roller is not blown into multiple pieces, rather it stays intact after the explosion, F_A is constant, and F_{IED} decreases over time ($F_{IED}=Ae^{-\beta t}$). Also, you need to consider this a two-step model: one modeling before the mine roller breaks from the vehicle, Model 1, and the other modeling after the mine roller breaks from the vehicle, Model 2. You should be given, or find through research, an initial condition for Model 1; the initial condition for Model 2 will be determined by the final condition from Model 1. We define: m=mass of mine roller [kilograms], x"=acceleration [meters/second^2], F_{IED} = force blast of the IED [Newtons], γ = drag of air[Newtons], x' = velocity [meters/second], F_a = force of mine roller attachment [Newtons], g = 9.807 meters/second^2, and T = threshold value when the mine roller breaks [seconds].

Model 1: $mx'' = F_{IED} - \gamma x' - mg - F_a, t \leq T$; Model 2: $mx'' = F_{IED} - \gamma x' - mg, t > T$

Solve

In Model 1, you are assuming that the mine roller is still attached to the vehicle after the IED explodes. In Model 2, you are assuming that the mine roller is detached from the vehicle. You could use Mathematica for both of these models or you could use the techniques necessary steps to solve a second order, linear, nonhomogeneous equation. For Model 1, you can research values that will help you find an initial condition and you can also use the information that your battle buddy found regarding the vertical displacement of the mine roller. The initial condition for Model 2 will be based upon the final condition from Model 1.

Interpret

To solve for the blast force of the IED, you must research your parameter values and use your battle buddy's information. Since Models 1 and 2 solve for the vertical displacement of the mine roller, you must substitute in your parameter values in order to be left with only F_{IED}. Another possibility would be to use a computer system to write a loop that increments the force each time until you reach the height found by your battle buddy.