**µProject 4**

**Bottom Line**

Another vehicle in Task Force Sinai has hit an IED. However, because of the blast-mitigating seats in the MRAP, no one in the vehicle was severely injured. I will attempt to explain why the blast-mitigating seats are preferable to traditional, static seats through the math modeling process in order to outfit all vehicles with these safer seats.

**Transform**

The key governing principle behind examining the seating is that it is a spring-mass system. Using this knowledge, I can construct a preliminary model with the following assumption: this model only covers IED blasts that come from underneath the vehicle with a force not powerful enough to completely penetrate the underside of the vehicle into the seat, the mass of the seat and the occupant will not change, and the damping system (blast mitigation system) will not be damaged. With this in mind, the model can be stated as such: $mx''(t) + \gamma x'(t) + kx(t) = g(t)$. This equation is the basic model for a spring-mass system, and the variables and coefficients are as follows: *m* is the mass of the seat and occupant, $\gamma$ is the drag/damping coefficient for the seat's blast mitigation system, *k* is the spring constant, *x(t), x'(t), x''(t)* are the position, velocity, and acceleration of the seat, respectively, and *g(t)* is a forcing function (external force) acting on the model. Furthermore, each one of these pieces can be broken down further for examination. Firstly, the mass of the occupant and the seat will remain constant throughout the event occurring. Secondly, the damping coefficient, $\gamma$, would be most beneficial if the system were overdamped. Like a car's suspension system, this would allow the occupant to move slower when the IED goes off. In a sense, the man's spine and neck would not be jerked around as violently. This means that $\gamma > 2\sqrt{km}$, this too would be a constant value determined by the oil and other fluids surrounding the spring housing. Finally, the spring constant, *k*, would be determined by the structural strength and composition of the metallic spring. The force of the IED imparted on the seat and its occupant can be considered an instantaneous function, essentially absorbed into the initial conditions for the seat's upward velocity, therefore allowing one to set the *g(t)* value equal to zero. This gives the homogeneous equation: $mx''(t) + \gamma x'(t) + kx(t) = 0$ for the position, acceleration, and velocity of the seat and its occupant.

**Solve**

So, in principle, this is a simple, homogeneous, second order differential equation with constant coefficients. One can solve this analytically by proposing an initial solution for *x*, determining another possible solution using factoring or the quadratic equation on the characteristic equation, checking to see that these pairs form a fundamental set, if necessary, using a Wronskian, solving for the constants associated with the format of the equation, whether it be algebraic or trigonometric by substituting in the initial values for the position and velocity of the seat and the occupant.

**Interpret**

Given the proposed model, one can calculate the acceleration and velocity and compare it to medical tables for their effects on human beings. In contrast, static seats do not have any sort of damping or spring to counteract the force of the IED blast upward, and the occupant's spinal cord likely acts as that spring-mass system. Because of this, static seats have a much higher likelihood of causing major traumatic injury to the occupant. Therefore, blast-mitigating seats should be implemented as much as is feasible.

**Bottom Line**

The impact of the blast on a soldier in an MRAP can be modeled through a second order damped and forced nonhomogeneous equation. This equation will model the acceleration felt by the person, allowing the force felt by the person to be calculated.

**Transform**

We need to create a model that relates the force acting upon the seat's shock system (the force of the IED) to the vibration (or vertical movement) of the seat. In order to do this, I used Newton's Second Law and Hooke's Law as the governing principles behind my model. Newton's law states that the sum of the external forces present is equal to the mass times the acceleration and Hooke's Law states that the force of the spring is equal to the negative of the length times the spring constant. Before applying this concept, I must assume that the total force of the IED is vertical and is applied directly below the seat being observed. I must also assume that the explosion acts over a given time ($t_E$)after which the forcing function is no longer applicable. Further, I assume that the spring system is critically damped, because this would get the seat back to equilibrium position as fast as possible.

After making these assumptions, I must set up an equation that takes into account the dependent variable position (x) and the independent variable time (t), and the effect of the mass, damping factor, spring constant, and the forcing function on x. Thus, I derived the equation $x" \equiv acceleration\ [\frac{m}{s^2}], m \equiv mass: seat + \ person\ [kg], F_{IED} \equiv the\ force\ of\ the\ IED\ blast\ [N], k \equiv$ $the\ spring\ constant\ [\frac{N}{kg}], \gamma \equiv the\ damping\ constant\ \left(2\sqrt{km}\right) when\ it\ is\ critically\ damped, F_G \equiv$ $the\ force\ of\ gravity\ acting\ upon\ the\ person$

$$x" = \frac{F_{IED} - kx - 2\sqrt{mk}x' - F_G}{m}, x(t_s) = .2032m, x'(t_s) = 0\frac{m}{s}$$

The initial conditions are as follows because we are given that at some time ($t_s$) we know that the displacement of the seat will be 8 in, or .2032m ("Metric Conversions"). When the seat is at this point its velocity will equal 0.

**Solve**

This is a second order, linear, nonhomogeneous equation that can be solved by first rearranging so that the dependent variable (x) is all on the left and then adding the solution ($y_c$) as if this were a homogenous equation, and the solution of this particular problem ($y_p$) to yield the general solution. This general solution will be an equation which describes the vertical distance (x) of the seat.

**Interpret**

In order to compare the utility of blast resistant seating to a vehicle without out, I will use the model equation I developed to compare the acceleration felt by the person in each of the two seats. The blast resistant seating will follow the model given while the seat without resistance will only include $x" = \frac{F_{IED} - F_G}{m}$. Thus, experimenters can test the two seats using a given force of the IED and can measure the mass of the seat/person system to calculate the acceleration felt by the person. The results will show that the acceleration is greatly decreased when there is blast resistant seating and the seat will return to equilibrium position much faster.

## Bottom Line

The effects of an IED explosion of a passenger in blast resistant seating can be modeled using a second order, linear, nonhomogeneous differential equation. This equation can then be solved and analyzed to determine the importance of such as system on mitigating casualties.

## Transform

Blast resistant seats are extremely important in mitigating traumatic effects on the human body during an explosion. These seats, such as those in an MRAP vehicle, operate on principles very similar to those of a hanging mass system. Springs are used as shock absorbers to slow the sudden change in acceleration caused by an explosion, which greatly reduces injuries. It is useful to have a simple visualization such as Figure 1 in order to better understand the basic layout of the system.

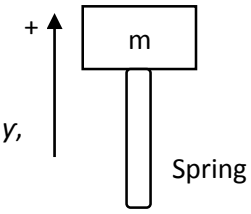


Figure 1: MRAP spring system

The mass of the passenger, *m*, is measured in kilograms. A certain amount of damping, *y*, measured in $\frac{kg}{s}$, is to be expected from the shock reducing system. A spring constant, *k*, measured in $\frac{kg}{s^2}$, will also be inherent in the system, and can be measured by calculating $\frac{mg}{L}$, where *L* is the length of the spring with no force pressing down minus the length of the spring with the force of the weight of the passenger.

Taking all of these variables together, a second order, linear, homogeneous differential equation can be created of the form $mx'' + yx' + kx = 0$. As this situation involves an IED explosion, an external force, *g(t)*, in Newtons, can be included to create a nonhomogeneous differential equation of the form $mx'' + yx' + kx = g(t)$, where x is the position of the seat (and passenger) at time *t* (in seconds). The function *g(t)* will be a function that decreases over time (such as $\frac{1}{t}$ or $e^{-t}$) as the force from the explosion decreases. Note that the "up" is the positive direction and "down" is the negative direction (see Figure 1).

## Solve

A second order, linear, nonhomogeneous equation can be solved in a few different ways, depending on the values of *m*, y, and *k*. If the discriminant expression $\sqrt{y^2 - 4mk}$ is greater than zero, the equation will have two real roots, $r_1$ and $r_2$, and can be expressed in the form $x(t) = C_1e^{r_1t} + C_2e^{r_2t} + x_p$. If the discriminant expression equals zero (the case of repeated roots) the solution can be expressed as $x(t) = C_1e^{rt} + C_2te^{rt} + x_p$. Finally, if the discriminant expression is less than zero, then a solution of the roots *a + bi* can be expressed as $x(t) = C_1e^{at}\cos(bt) + C_2e^{at}\sin(bt) + x_p$. In each of these cases, $x_p$ can be solved depending on *g(t)* using the method of undetermined coefficients. Similarly, given initial conditions of the position and velocity of the system, the constants $C_1$ and $C_2$ can also be determined.

## Interpret

The model that has been created attempts to duplicate the position of a passenger during an IED explosion provided they have an MRAP system. This system absorbs force from the movement of the vehicle, slowing the acceleration of the passenger and mitigating injuries. Without such a system, the passenger would accelerate at a rate equal to the acceleration of the vehicle. An experiment could be performed that applies a known blast force to a crash test dummy in a vehicle with and without the MRAP system. The forces applied to the dummies could then be measured, compared, and analyzed to determine the utility of the MRAP system.