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STUDENT VERSION
TWO SPRINGS ONE MASS FIXED ENDS

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Abstract: Students build a model of a two spring, single mass with fixed end configuration and then plot solutions to experience the motion.

SCENARIO DESCRIPTION

We consider two springs (coils - top one thin as it has smaller spring constant and bottom one thicker as it has larger spring constant) and a mass (plate) in a vertical mode configuration as depicted in Figure 1. We illustrate the configuration at two different positions in Figure 1.

In Figure 1 the figure on the left is when the spring system is in static equilibrium, i.e. at rest, while the figure on the right has the mass displaced vertically down from static equilibrium.

In addition to the mass, m in grams, Spring 1 has the following associated parameters - spring constant k_1 (in units of dynes/cm); resistance c_1 in units (dyne·sec/cm or dyne/(cm/sec)); while Spring 2 has the following associated parameters - spring constant k_2 (in units of dynes/cm); resistance c_2 in units (dyne·sec/cm or dyne/(cm/sec)).

We shall measure distance from the static equilibrium as $y(t)$ cm at time t sec. We shall adopt the convention that when $y(t) > 0$ the mass is below the static equilibrium and when $y'(t) > 0$ the mass is moving downward. In the second picture, then, the displacement is positive. Do you see that? Furthermore, positive velocity will occur when the mass is traveling downward. These, of course, are arbitrary assignments, but we use them throughout to be consistent.

Finally, we shall need some initial conditions, say $y(0) = y_0$ and $y'(0) = v_0$. Let us assume $v_0 = 0$, i.e. initially we merely displace the mass y_0 cm from its static equilibrium with zero velocity, simply releasing it.

Here is your set of tasks:

1. Build a mathematical model for the displacement $y(t)$ of the mass in this configuration, using Newton's Second Law and a Free Body Diagram of the forces acting on the mass, m, and the

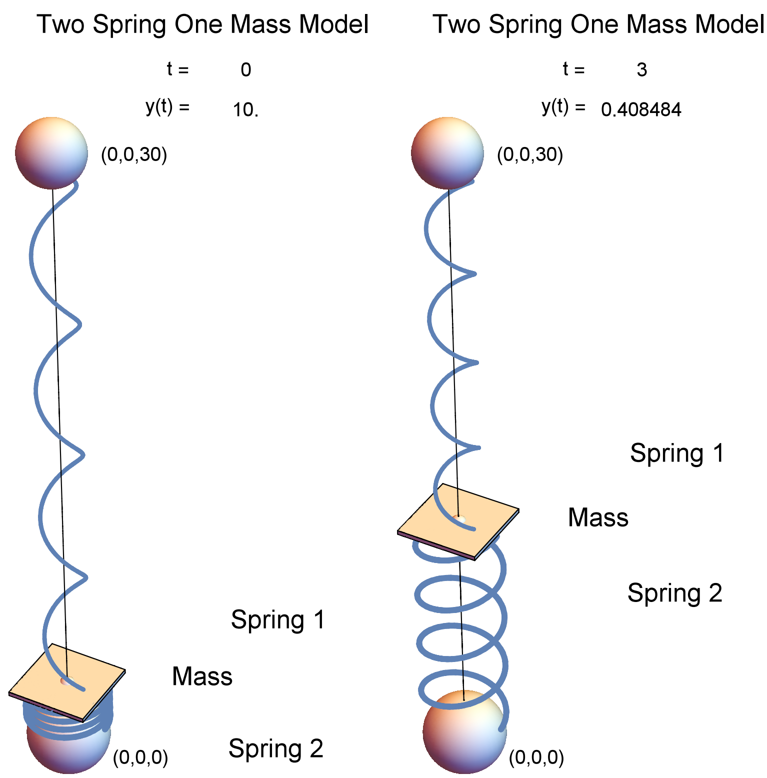

Figure 1. Two springs with fixed ends and one mass in two positions. On the left the view is when the spring system is in static equilibrium, i.e. at rest, while on the right is when the mass is displaced vertically down from static equilibrium.

named parameters above. You should come up with a second order differential equation which we shall calibrate before moving on.

2. Confirm that from a Free Body Diagram of the forces acting on the mass, m, the proper differential equation model is:

$$my''(t) = -(c_1 + c_2)y'(t) - (k_1 + k_2)y(t) \tag{1}$$

For ease in constructing (1) consider the mass as displaced below the static equilibrium and moving downward.

3. Convert the second order differential equation into a system of linear first order differential equations using the "key": $x(t) = y'(t)$.

4. Use these values for the parameters in the problem, $m = 10$, $k_1 = 20$, $k_2 = 30$, $c_1 = 10$, $c_2 = 10$, $y_0 = 10$, and $v_0 = 0$. Solve this system using an eigenvalue and eigenvector approach completely!
5. Using some tool, obtain a plot of the displacement, $y(t)$, over the time interval $[0, 6]$ sec.
6. Compare the solutions to the two approaches.
7. Here are some further questions to consider. Take on as many as assigned.
   (a) What would happen if we DECREASED both c_1 and c_2 (equally)?
   (b) What if both c_1 and c_2 were 0? Could they be?
   (c) What would happen if we DECREASED both k_1 and k_2
   (d) What would happen if we changed y_0? Say making it negative?
   (e) What would happen if we changed the mass, m?
   (f) What would happen if we used a value of $y'(0)$ not 0?