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STUDENT VERSION

Thanos Population Dynamics: Interacting Species

Sarah Patterson | Blain Patterson
Applied Mathematics
Virginia Military Institute
Lexington VA 24450 USA

Abstract: In the end of the "Avengers Infinity War," the villain Thanos snaps his fingers and turns half of all living creatures to dust with the hope of restoring balance to the natural world. How does this affect the long term behavior of various species? Investigate the validity of his claim by modeling various population dynamics such as competing species and predator-prey.

SCENARIO DESCRIPTION

In the 2018 blockbuster, Marvel Studios' "Avengers: Infinity War," the villain Thanos snaps his fingers and turns half of all living creatures in the universe to dust [1]. He was concerned that overpopulation on a planet would eventually lead to the suffering and extinction of the entire population. This is evident in the following quote from Thanos.

"Little one, it's a simple calculus. This universe is finite, its resources finite. If life is left unchecked, life will cease to exist. It needs correction."

1. There is a bit to unpack in Thanos's quote. What are some of the assumptions that Thanos is making?

In this activity, we will investigate the validity of Thanos's claims using mathematical models for population dynamics.

2. Given a population model, how could we simulate Thanos's snap that instantly removes half the population from existence? Would this change the growth rate? Would this change the initial conditions?

Initially, we may want to investigate how Thanos's plan affects the population dynamics of a single species. However, species do not live in a vacuum and the presence of other species can have a large effect on the population. In this activity, we will look at two types of species interactions and what would happen to these species under Thanos's plan.

3. Suppose that a planet that had only two species, how would Thanos's snap affect the two populations?

Competing Species

In many cases, two populations P_1 and P_2 will compete for the same resources, which can result in some interesting population dynamics. If we assume the absence of species P_2, the species P_1 grows according to the logistic equation. However, the presence of species P_2 will inherently lower the growth rate of the other species. This results in the following system of differential equations.

$$\frac{dP_1}{dt} = k_1 P_1 (1 - \frac{P_1}{L_1}) - c_1 P_1 P_2$$
$$\frac{dP_2}{dt} = k_2 P_2 (1 - \frac{P_2}{L_2}) - c_2 P_1 P_2,$$

where k_1 and k_2 represent the growth rate of population P_1 and P_2, L_1 and L_2 represent the carrying capacity of the population of P_1 and P_2, and c_1 and c_2 represent the competition strength of each population, respectively.

4. What effect does each parameter have on the model?

5. Investigate various population dynamics by researching species and choosing reasonable values for each parameter.

6. Model Thanos's snap which removes half of all living creatures. Observe the effects on the long term behavior compared to your original model. Determine any equilibrium values. Is the coexistence of both species possible? Will the extinction of one of the species occur?

Predator-Prey

Rather than competing for limited resources, other populations interact as predator and prey. In this case, we assume that the prey is always able to find ample food and that the food supply of the predatory is entirely dependent on the population of the prey. This results in the following system of differential equations.

$$\frac{dP_1}{dt} = aP_1 - bP_1P_2$$
$$\frac{dP_1}{dt} = cP_1P_2 - dP_2,$$

where P_1 is the number of prey, P_2 is the number of predators.

7. What effect do the parameters *a,b, c, and d* have on the model?

8. Investigate various population dynamics by researching species and choosing reasonable values for each parameter.

9. Model Thanos's snap which removes half of all living creatures. Observe the effects on the long term behavior compared to your original model. Determine any equilibrium values. Is the coexistence of both species possible? Will the extinction of one of the species occur?

10. Why did you choose to implement Thanos's snap at the time that you did? Implement the snap at different points in time. Does the time that the snap is implemented affect Thanos's plan?

Homework

Populations rarely only interact with one other species. How would you model a three species interaction where one is the predator and two are prey that compete for one resource? How would this more complicated scenario play out under Thanos's plan?

REFERENCES

[1] *Avengers: Infinity War*. Directed by Anthony Russo. Burbank: Marvel, 2018.