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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
From Data to Differential Equations

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Abstract: Students use their knowledge of second-order linear differential equations in conjunction with physical intuition of spring-mass systems to estimate the damping coefficient and spring constant from data. The data is presented as *total distance traveled* instead of displacement so the students must rely on previous calculus knowledge as well as knowledge of differential equations and numerical differentiation.

SCENARIO DESCRIPTION

A motion sensor was set up to measure the motion in a spring-mass system, but something went wrong and the motion sensor measured the total distance traveled instead of simply measuring the distance from the sensor. The experiment was conducted three times (same spring and same mass) with different initial conditions. The total distance traveled is given as the data sets in the Excel spreadsheet titled *SpringMassData.xlsx*. The conditions for the three experiments were:

Data Set #1: Initial position: $y(0) = 1$. Initial velocity: $y'(0) = 0$.

Data Set #2: Initial position: $y(0) = 0.5$. Initial velocity: $y'(0) = 1$.

Data Set #3: Initial position: $y(0) = -0.75$. Initial velocity: $y'(0) = -2.5$.

Experimental Setup:

- The sensor gathered data at a rate of 20 samples per second.
- The experiment was run for 5 seconds.
- $y(t)$ is the distance from the equilibrium position of the spring-mass system. Positive values of $y(t)$ indicate that the mass was above the equilibrium position. Negative values of $y(t)$ indicate that the mass was below the equilibrium position.

- There is some noise in the data.

Your Task: Use the data in the associated Excel spreadsheet to determine the governing differential equation along with estimates of the parameters. Include all necessary plots and mathematics.
Recall:

- For an un-driven spring-mass system you can either use

$$my''(t) + by'(t) + ky(t) = 0$$

  where m is the mass, b is the damping coefficient, and k is the spring constant or you can divide through by the mass and use

$$y''(t) + \delta y'(t) + \kappa y(t) = 0$$

  where $\delta = b/m$ and $\kappa = k/m$.

- Recall that if $y(t)$ is displacement and $v(t) = y'(t)$ is velocity then the total distance traveled is given by the function

$$d(t) = \int_0^t |v(\tau)|\, d\tau$$