

Euler's Method Activity Sheet

Suppose that $y = f(t)$ satisfies the following conditions:

$$\frac{dy}{dt} = \cos t \text{ and } f(0) = 0.$$

1. Verify that $y = \sin t$ is the solution to the above initial value problem.
2. Fill out the following tables using Euler's method and the appropriate given value of Δt .

$$\frac{dy}{dt} = f(y, t) \text{ with } y(t_0) = y_0$$

$$y_{n+1} = y_n + \Delta t \cdot f(y_n, t_n)$$

t	$\sin t$	y_n	$ \sin t - y_n $

Table 1. $\Delta t = \frac{\pi}{2}$

t	$\sin t$	y_n	$ \sin t - y_n $

Table 2. $\Delta t = 0.1$

3. Explain what the column labelled $|\sin t - y_n|$ represents. How does it differ between the two values of Δt ?
4. Plot the points (t, y_n) and $(t, \sin t)$ from Table 1 and 2 on the next page. Be sure to label the axes appropriately.
5. Can you explain your observations in 3 using the above tables and plots?
6. What could be done to achieve a better numerical approximation to the solution of the original initial value problem?

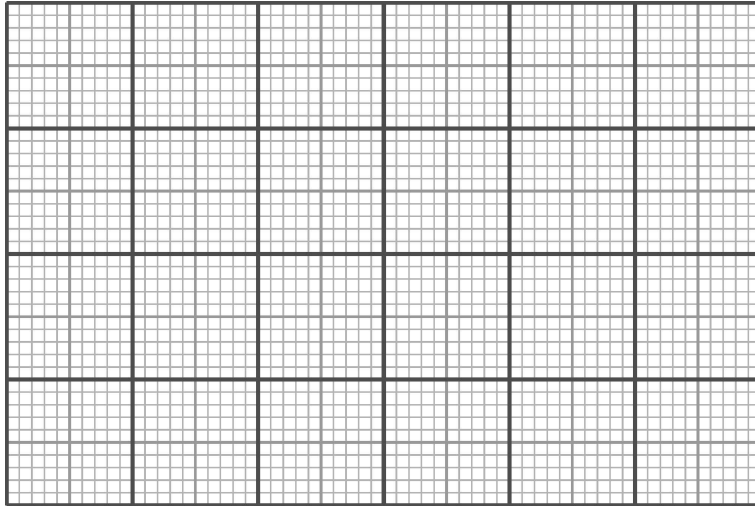


Figure 1. $\Delta t = \frac{\pi}{2}$.

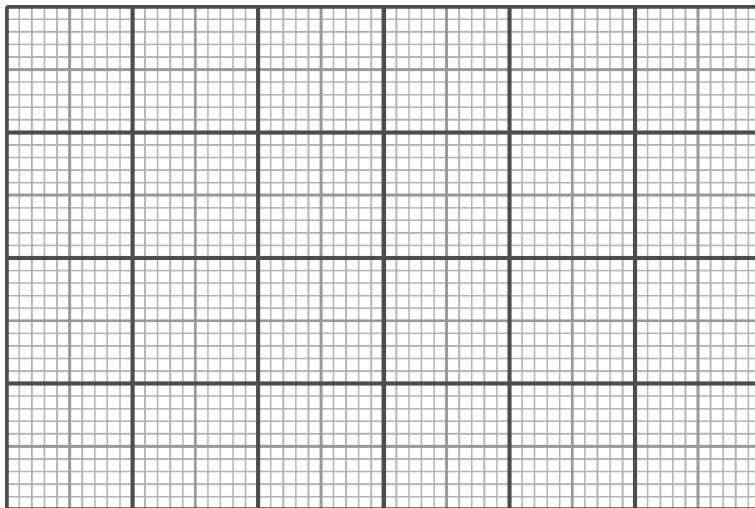


Figure 2. $\Delta t = 0.1$.