

## STUDENT VERSION DESIGNING A FAIR HOME RUN STADIUM

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**Abstract:** For a given playing field outline how high must the outfield fence be at each point in order to make a homerun equally likely in all fair directions?

## SCENARIO DESCRIPTION

Consider a baseball field with prescribed irregular outfield and fence configuration.

Suppose a player can hit a home run which just clears the fence at one given point if the ball leaving the bat (2 feet off the ground) has initial velocity v0 and launch angle at 45 degrees. Assume no wind resistance. Redesign the fences so that this player (and hence any player) has the same chance of hitting a home run to anywhere in the outfield (i.e. in the outfield and fair!)

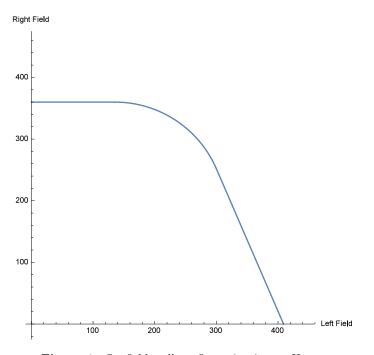
We consider Hypothetical Stadium (HS) with the following outfield wall configuration. See Figure (1). Home plate is at (0,0); left field foul pole is straight up the positive y-axis; right field foul pole is straight down the positive x-axis.

The left field outfield fence runs parallel to first base line for 135 ft at a distance of 360 ft from home plate.

The right field outfield fence is a straight line from right field foul line at (409, 0) to the point (300, 252).

Center field outfield fence is a circular wall going through deepest center field point out at  $(200\sqrt{2}, 200\sqrt{2})$ , point LC = (135, 360) in left center, and point RC = (300, 252) in right center.

- a) Write equations which completely describe the outfield fences and plot them with home plate at (0,0).
- b) Determine the farthest point of the outfield fence from home plate, measured along the ground to the base of the fence.



**Figure 1.** Outfield wall configuration in our Hypothetical Stadium. Home plate is at the origin (0,0).

- c) Place a 10 ft high fence at this farthest point found in (b) and determine the initial velocity a hitter must impart to a ball hit 2 ft high over home plate to get it to just clear the 10 ft high fence here. We shall assume there is no air resistance. Hint: It is known (and you should be able to prove it is true!) that a hitter should always try to launch the ball from the bat at a 45° angle with the horizontal for maximum range.
- d) Now determine for every other point at the base of the outfield fence just how high the fence must be so that this player can hit a home run by launching a ball from his bat 2 feet above home plate with exactly the same initial velocity at a 45 degree angle with the horizontal.
- e) Discuss the reasonableness of your answers. Would baseball "buy" your idea?

How would resistance alter (if at all) the shape of your wall?

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