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STUDENT VERSION
Spring Costs

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Abstract: We assume students are familiar with overdamping and underdamping of a spring-mass-dashpot system. Students will apply this knowledge to model the interplay between spring constant, tolerance, and cost.

SCENARIO DESCRIPTION

Stiffness of a spring is given by the "restoration constant", k, in units of Newtons per meter, or N/m. Time is measured in seconds (s). The greater the stiffness, the more expensive the spring. Let us assume a cost, in dollars, for a 0.2 m long spring of $\text{Cost}(k) = 13.70 + .01k^2$. 5 Now suppose we are designing the following spring mass dashpot system:

$$1.2 \cdot y''(t) + 46.4 \cdot y'(t) + k \cdot y(t) = 0\,, \quad y(0) = 0.4\,, \quad y'(0) = 0\,. \tag{1}$$

We can assume the initial position of the spring mass is 0.4 m from the static equilibrium and its initial velocity is 0 m/s in all situations.

i) Find the values of k which make the system overdamped.
ii) Suppose in the case of the overdamped configuration we wish to bring the mass to within 0.1 m of the static equilibrium in the first 0.5 s of motion. We call this 0.1 m the *tolerance*. What k value would we use to do this? How much would such a spring cost us?
iii) What kind of spring performance can we get if we are willing to spend \$600 on the spring while meeting the specifications of (ii)?
iv) It is conjectured that for a reduction to x% of the tolerance, i.e. x% of tolerance of 0.1 m close to the static equilibrium, there is an x% increase in price. This means that when $x = 8$ we are comparing costs of a spring which will get us to within 0.1 m of the static equilibrium in 0.5

s to a spring that will get us to within $0.01 \cdot x = 0.08$ m of the static equilibrium in 0.5 s, a reduction in tolerance to 80% of original tolerance of 0.1 m.

Prove or disprove this conjecture, and provide data on costs for springs in terms of k which will get us to within $z/100$ m of the static equilibrium in 0.5 s for $z = 1, 2, 3, \ldots, 9$.

Is there a model for this data you can offer?

v) Finally, can you render a relationship and an approach for finding the minimum cost spring which meets the two criteria (1) drops below distance a meters from the static equilibrium and (2) does so within the time of b seconds after release? Ideally, given the specifications a and b what is the k to produce the cheapest cost $\mathrm{Cost}(k) = 13.70 + .01k^2$ spring and what is that cost?