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SIMIODE
A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS
& OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
BUOYANCY

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Abstract: We offer data from a physical experiment in which the depth (from equilibrium value) of a container in water is measured 20 times per second and ask students to build a model of buoyancy based on Newton's Second Law of Motion and a Free Body Diagram. We ask students to estimate the parameters in their model and validate their model against the data.

SCENARIO DESCRIPTION

We have all experienced buoyancy, be it with our little rubber ducky when we were in the tub or floating in the ocean or bobbing for apples (they did not sink). Can we model a bouncing or bobbing object in water though?

Let us consider an object floating in water. We press it down and it bobs back up, and then down, and then up, and then down, etc. until it comes to rest in the wavy water around it, caused by its own up and down motion.

The container depicted in Figure 1 is an inverted container in the shape of a frustum of a right circular cylinder. It has a cover and contains some rocks with some water on the rocks.

In Figure 2 we see an apparatus in which a blue topped plastic prune container with cover and some rocks and some water on those rocks is set bobbing up and down in a five gallon bucket of water. Above the container we see a motion detector held by a clamp with its detection face pointing down. The GoMotion motion detector (Vernier Software and Technology) is held in a clamp above the bouncing container and collects data on the distance between the detector and the top surface of the container every 20^{th} of a second. This data is then collected in Vernier's *Logger Pro* software, transferred to *EXCEL*. From here it may be used in any application such as *Maple*, *SAGE*, or *Mathematica*.

We set the motion detector to on, push the container down into the water a ways, and release it. After a few seconds we stop the motion detector and examine the data output. Figure 3 is

Figure 1. Container partially filled with small stones for weight and used in the collection of the data.

anexample of the recorded data. We see on the left the time and position information and on the right the plots of position and velocity. The "nice" oscillating data in the 6 to 9 second range are what interest us. The other motion data come from the movement of the experimenter's hand and the placement of the container, ready to start the data gathering.

Model Building

Let us give some ideas for building a model to track the height of the point which sits at the water line when the container is at static equilibrium. We will call this distance $y(t)$ (in meters) at time t in seconds. We will obtain this distance from collected data set. For consistency we will use the convention that when the point is above the water line then $y(t) > 0$ and when the point is below the water line then $y(t) < 0$. Of course when this point is at the water line either in static equilibrium or just passing through on its bobbing mission $y(t) = 0$.

We seek a model which will permit us to describe $y(t)$ for $t \geq 0$. Let us recall Newton's Second Law of Motion which says that the mass times the acceleration a body experiences is equal to the sum of the external forces acting upon that mass. We first recall that at static equilibrium the downward force of the mass of the container times gravity (called the container's weight) is offset by the buoyant force equal to the liquid displaced (Archimedes' Principle) and so we need not consider these two force in the formulation of an equation of motion using Newton's Second Law of Motion.

1) In order to help us realize the forces acting upon the container let us draw a Free Body Diagram and articulate the external forces. Do that now and consult with your classmates and your teacher to come to a consensus.

Figure 2. Apparatus for collecting data on the vertical motion of a floating container.

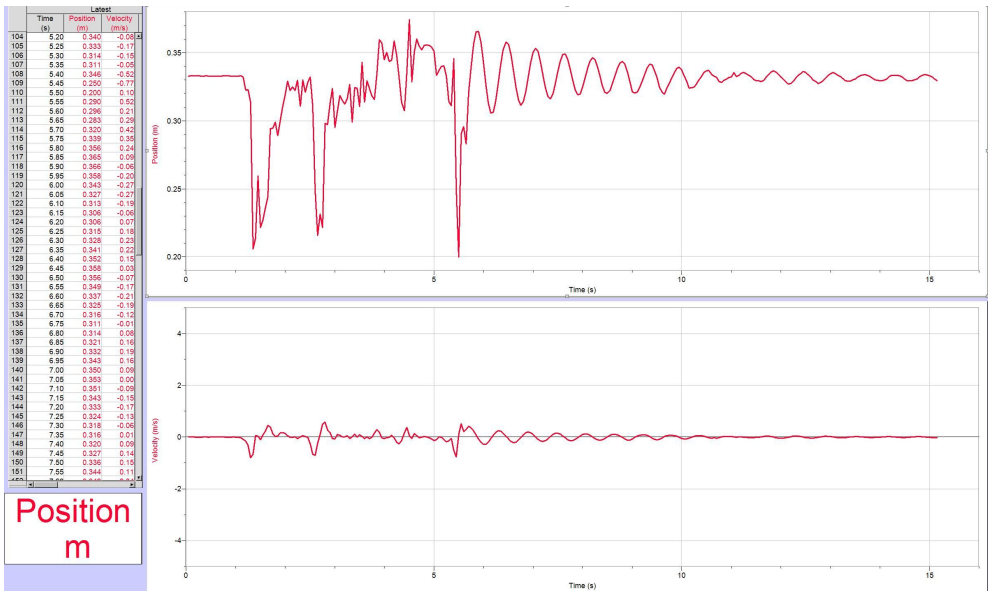

Figure 3. A sample of data collected from the GoMotion motion detector.

2) After you have a reasoned Free Body Diagram, have identified all issues, and established conventions on directions of motion and force (see our discussion above on $y(t)$) construct a differential equation model by filling in the right hand side of (1)

$$my''(t) = \tag{1}$$

where $m = 0.51329$ kg is the mass of our container.

3) Solve (1) in general.

4) If you had observed data on $y(t)$ at time t for a number of points in the bobbing of the container explain how you would go about estimating the parameters you have introduced on the right had side of your (1).

5) Use the observations in either the *EXCEL* spreadsheet 3-006-S-EXCEL-Buoyancy-StudentVersion (where there are two sets of observations with instructions) or in the *Mathematica* notebook 3-006-S-Mma-Buoyancy-StudentVersion (where there is one set of observations with instructions) - you pick one data set and perform your analysis which you outlined in activity (4).

6) Offer up several ways you could improve this modeling of a buoyancy phenomenon.