www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
SPRING MASS DATA ANALYSIS

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA
BrianWinkel@simiode.org

Abstract: We offer data on the position of a mass at the end of a spring over time where the spring mass configuration has additional damping due to taped flat index cards at the bottom of the mass. The general modeling of a spring mass configuration and the estimation of parameters form the core of this activity.

SCENARIO DESCRIPTION

We study a spring mass system as depicted in Figure 1 with a spring attached to a horizontal rod on a ring stand and a Vernier Motion Detector under the mass to capture its displacement from a static equilibrium value at time intervals of 0.02 seconds over a time interval of about 30 seconds. The motion detector is turned on and the spring is stretched downward and released. We collect the data and ignore (edit out) data from hand placement until a reasonable point in the motion when the oscillation is clear and clean. That cleaned up data is moved to an Excel spreadsheet and a Mathematica notebook. Figure 1 depicts the apparatus with the mass having no additional resistance material in the form of taped index cars on the mass. Figure 2 shows the 200 g mass with index cards taped to its base to offer more resistance. This makes for a total mass of 200.20626 g.

PRELUDE

Consider the abstract Free Body Diagram for the mass at the end of the spring in Figure 3. We will presume that the mass is settled into a static equilibrium in which the force due to gravity is countered by the restoring force of the spring to contract or expend depending upon whether the mass is below or above this static equilibrium point, respectively, and hence, pull up or push down, respectively, the mass. Suppose we let $y(t)$ be the distance the mass is from static equilibrium and $y(t) > 0$ for when the mass is below the static equilibrium, i.e. when the spring is extended, and $y(t) < 0$ for when the mass is above the static equilibrium, i.e. when the spring is compressed,

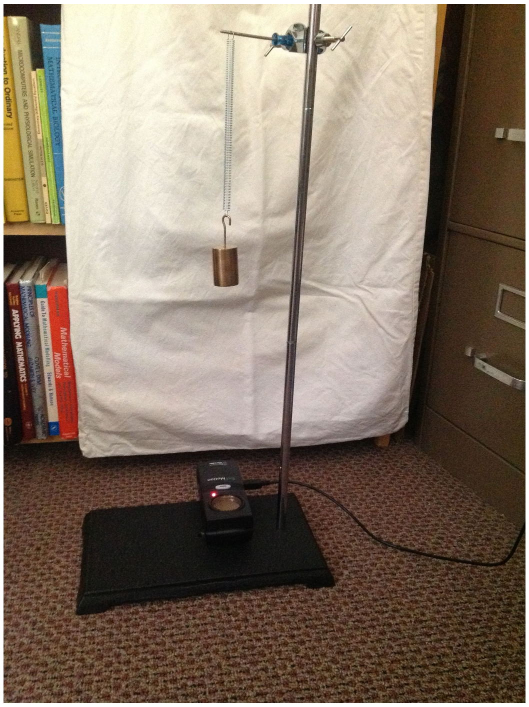

Figure 1. The apparatus setup needed to collect data on spring mass motion.

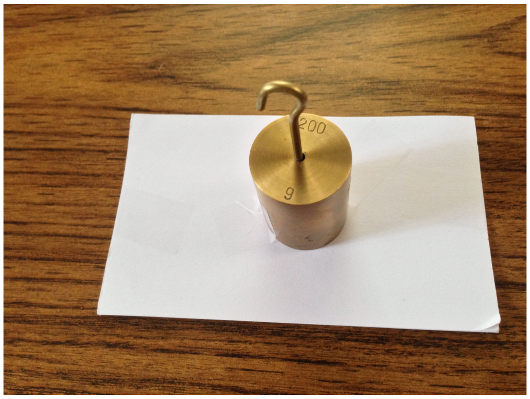

Figure 2. The mass used in the experiment with 4 index cards taped to the base.

Newton's Second Law states that the sum of the forces acting on a body are equal to the product of the body's mass and acceleration. See (1) below in preparation for building a differential equation model using this Law. We need to identify these acting forces. Remember the force due to gravity has been offset by the restoring force of the spring to contract or expand when at its static equilibrium or rest. Now the two forces we need to consider are (1) the force to restore the spring to the static equilibrium value and (2) the force of resistance to motion due to friction in the spring and resistance.

The restoring force of the spring to restore (contract or expand) the spring can be modeled by Hooke's Law which says that "...that the force needed to extend or compress a spring by some distance is proportional to that distance. This is a constant factor characteristic of the spring and is often referred to as its *stiffness*. The law is named after 17th century British physicist Robert Hooke."[1]

The force of resistance is often just proportional to the velocity of the mass and is due to internal friction in the spring and external resistance due to the medium, in this case air with flat index card surface in opposition to the motion.

We will be modeling a spring mass system and seeing how well it describes data taken from an experiment.

Activity 1

Draw these acting forces in Figure 3 and incorporate them into (1) according to Newton's Second Law of Motion.

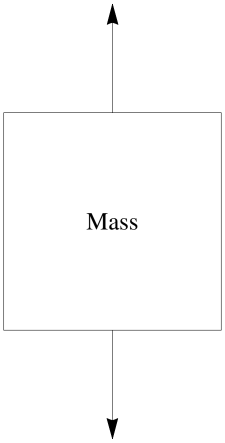

Figure 3. Base diagram for constructing a Free Body Diagram showing the forces on a mass at static equilibrium, excluding gravity and its counteracting restoring force of the spring to contract or expand depending upon whether the mass is below or above the static equilibrium point.

$$m\,y''(t) = \underline{\hspace{5cm}} \tag{1}$$

Activity 2

Use a mass of 206.26 g (or 0.20626 kg) in your spring mass model (1) and some "toy" values for the parameters in your terms of Activity 1, e.g., $c = .01$ N/(m/s) in your resistance term $c\,y'(t)$ and $k = 14$ N/m in your restoration term $k\,y(t)$. Here the parameter k is called the *spring constant*.

Later we shall use data from an experiment for which the mass is $m = 206.26$ g where $y(0) = 0.0419$ m and $y'(0) = 0.148$ m/s. So you can use these as initial position and velocity in this Activity. These initial conditions mean the spring has been extended downward by 0.0419 m and has an initial velocity of 0.148 m/s.

Solve the differential equation model you built from (1) with the above values and plot the distance of displacement over the first 20 seconds of motion.

Describe this motion and explain why it is reasonable (or not!) Did it meet your expectations?

Activity 3

Take your model from Activity 2 and tweak one (only one at a time) of the parameters, e.g., mass m, resistance parameter c, or spring constant k. Keep all else the same. Perhaps you can describe some phenomenon related to the motion in terms of changing value of your parameter. Then report the results of your motion in response to these changes in the parameter.

Activity 4

In an Excel spreadsheet, 3-1-S-SpringMassDataAnalysis-StudentVersion.xls, and a Mathematica notebook, 3-1-T-Mma-SpringMassDataAnalysis-StudentVersion.nb, we offer up good data we collected for the position of the mass at the end of an oscillating spring at times $t = 0$ to $t = 29.80$ s in time increments of 0.02 s.

Using this data and your model estimate the parameters c and k and verify whether or not your differential equation model you obtained from (1) is actually a good model for this phenomenon. Be sure you state your criteria for "good."

NOTES FOR TEACHERS

Just how the teacher introduces the model depends upon what the students have done previous to this. If this is the first time that spring-mass configuration has been studied, then the Free Body Diagram approach is important, otherwise one can jump directly to the differential equation, (2),

$$my''(t) + cy'(t) + ky(t) = 0\,, \qquad y(0) = y_0\,, \qquad y'(0) = v_0\,. \tag{2}$$

Of course, in this offering we "guide" the students to this model in Activity 2 by suggestion. Thus, while they may offer their own model we strongly encourage them to use the established model. We have gone with student models and this is instructive as they see the consequences and issues, e.g., making resistance term proportional to the velocity squared or just making the restoration force constant.

The suggestion of playing with one of the parameters permits students to numerically study changes in these parameters as do the graphs.

The actual parameter estimation requested in Activity 4 could be quite demanding of students, but if the minimization of the sum of square errors has been in play already then this estimating of TWO parameters could be a nice extension from one parameter estimation.

We offer extensive analyses in the Mathematica notebook, 3-1-T-Mma-SpringMassDataAnalysis-TeacherVersion.nb and the corresponding .pdf file, 3-1-T-Mma-SpringMassDataAnalysis-TeacherVersion.pdf, for those who do not use Mathematica.

REFERENCES

[1] Wikipedia contributors. 2014. Hooke's law. Wikipedia, The Free Encyclopedia. http://en.wikipedia.org/w/index.php?title=Hooke%27s_law&oldid=628866252.