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STUDENT VERSION
Empirical Models for Tropical Storm Windspeeds After Landfall

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Abstract: We model the decay of tropical cyclone winds once a storm makes landfall. We use data from two recent storms from the National Hurricane Center to estimate parameters emanating from a differential equation using a first order exponential decay model. This modeling scenario is suitable for students who have experience with separable equations.

SCENARIO DESCRIPTION

Hurricane Forecasting Models

Due to the potential tragic nature of tropical systems, there is a need for the scientific understanding and modeling of these complicated phenomena in order to reduce unwanted destruction and prevent unnecessary deaths. Hurricanes are large, swirling storms with winds of 119 kilometers per hour (74 mph) or higher and are usually characterized by a low-pressure center, a closed low-level atmospheric circulation, strong winds, and a spiral arrangement of thunderstorms that produce heavy rain. Coastal damage may be caused by strong winds and rain, high waves (due to winds), storm surges (due to severe pressure changes), and the potential of spawning tornadoes. Tropical storms also draw in air from a large area—which can be a vast area for the most severe storms—and concentrate the precipitation of the water content in that air (made up from atmospheric moisture and moisture evaporated from water) into a much smaller area. This continual replacement of moisture-bearing air by new moisture-bearing air after its moisture has fallen as rain, may cause extremely heavy rain and river flooding up to 25 miles inland from the coastline, far beyond the amount of water that the local atmosphere holds at any one time.

The National Hurricane Center (NHC) uses forecast models (any objective tool used to generate a prediction of a future event) in order to predict and prepare for tropical storms. Specifically, they are used as a way of guiding the NHC in their preparation of official storm track and intensity forecasts. The forecast models can take on a variety of different forms of varying complexity which seek to provide predictions for the storm's track, intensity, and/or wind radii. These methods use a variety of mathematical (and statistical) techniques such as multiple regression, dynamical systems and logistic growth. Many of these models focus on one particular aspect of the storm. For instance, the LGEM, or *logistic growth equation model* is a statistical intensity model, which uses ocean heat content to predict the intensity of a tropical storm.

In an effort to better understand and predict the path and intensity of a land-falling tropical system, we propose the development of a model for predicting the maximum sustained wind speed of landfalling tropical cyclones.

Problem 1

The goal is to come up with a mathematical model for predicting the maximum sustained wind speed (MSWS) of a land-falling tropical cyclone. To do so, we must consider some simplifying assumptions. What are some assumptions that we can make which will aid us in developing the model? Below are some considerations. Which assumptions/considerations do you believe are most important to the development of the model?

- sea temperatures
- dry air
- vertical wind shear (i.e. variation in wind velocity occurring along a direction at right angles to the wind's direction and tending to exert a turning force.)
- size of storm
- location of storm

Problem 2

To build the simplest model possible, we only assume that upon making landfall, tropical cyclone winds decay rapidly and in proportion to the current strength of the cyclone. See [2] for more information. Let $v(t)$ (in miles per hour) denote the MSWS at time t (in hours) after the hurricane has made landfall. With the assumption given at the beginning of this problem, develop a differential equation which models this decay. Let α denote the proportionality constant.

Problem 3

Solve the differential equation in Problem 2. Let v_0 denote the initial MSWS when the tropical cyclone makes landfall (corresponding to $t = 0$). Compute $\lim_{t\to\infty} v(t)$. What do you observe about your limit? Realistically, as time progresses, it has been observed that the MSWS for a tropical cyclone decreases to a nonzero limiting wind speed. Equipped with this knowledge, adjust your model in problem 2 so that $\lim_{t\to\infty} v(t) = v_b$, where v_b denotes this limiting nonzero wind speed.

Problem 4

The solution that you have obtained in Problem 3 is a two-parameter solution in the sense that α (the decay constant) and v_b, the limiting wind speed, must be estimated from data. To this extent, consider the following data obtained from the National Hurricane Center, [3], for Hurricane Irma, an extremely powerful and catastrophic Cape Verde hurricane, which made landfall in the continental United States over Florida in 2017. See Figure 1 for Hurricane Irma's track.

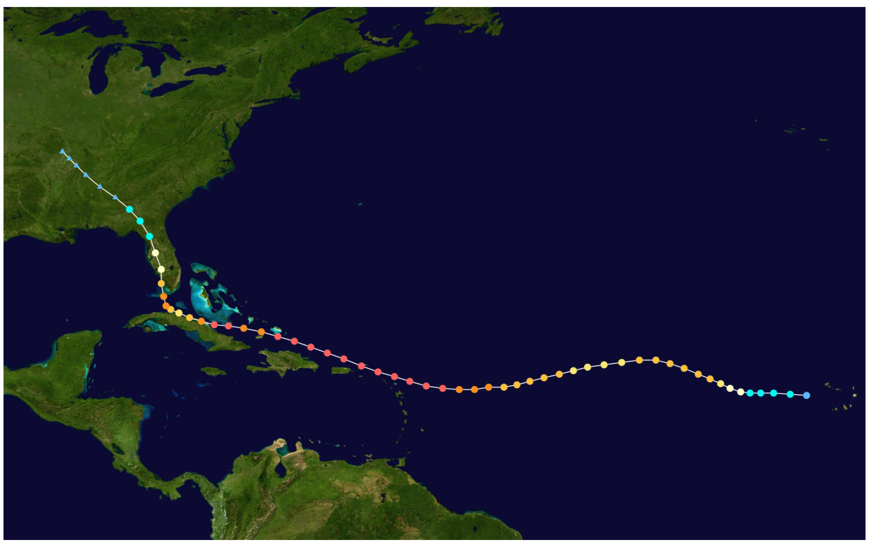

Figure 1. The track of Hurricane Irma [6]

We wish to estimate α and v_b from this data. Using Excel worksheet *hurricane.xlsx* along with this exercise, we can obtain the scatter plot in Figure 2 of the wind speeds at different times after landfall.

We will use the Solver package in Excel to find optimal values for v_b and α that best predicts the MSWS of Hurricane Irma. To use Solver in Excel for non-linear curve fitting, begin by creating a spreadsheet similar to the one in Figure 3.

Your solution should have the form

$$v(t) = A + B * e^{-Ct}. \tag{1}$$

Using the data as a guide, try to determine values for v_0, v_b and α so that the computed wind speeds match the data. What factors did you use to decide how to find these *optimal* values? In

time after landfall (in hours)	**MSWS (in mph)**	**time after landfall (in hours)**	**MSWS (in mph)**
0	130	12	100
1	130	13	100
2	130	14	100
3	120	15	85
4	120	18	75
5	115	21	70
6	110	24	65
7	110	27	60
8	110	30	50
9	105	33	45
10	105	36	35
11	105	39	25

Table 1. Irma's MSWS after making landfall in Florida. [3]

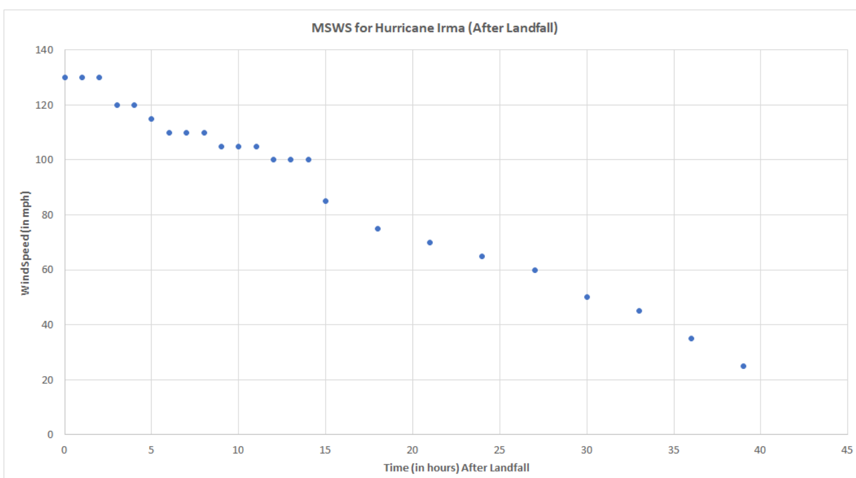

Figure 2. Scatter Plot for the wind speeds of Irma After Landfall

Figure 3, column C is the predicted MSWS based upon the initial guesses of v_0, v_b and α. The formula entered into cell C2 will depend on your solution obtained in Problem 3. For instance, if the computed solution were

$$v(t) = C + A(1 - e^{Bt}),$$

we would enter the corresponding formula:

	A	B	C	D	E	F	G	H
1	speed in mph	Time (in hours)	predicted speed in mph	chi squared		**A**	1	
2	130	0	3	16129		**B**	2	
3	130	1	3.060909068	16113.53281		**C**	0.03	
4	130	2	3.123673093	16097.60233		**X^2**	16097.6	
5	120	3	3.188348567	13644.96191				
6	120	4	3.254993703	13629.3965				
7	115	5	3.323668485	12471.60302				
8	110	6	3.394434726	11364.74655				
9	110	7	3.46735612	11349.20421				
10	110	8	3.542498301	11333.19967				
11	105	9	3.619928901	10277.91882				
12	105	10	3.699717615	10261.74721				
13	105	11	3.781936257	10245.09643				
14	100	12	3.866658829	9241.619285				
15	100	13	3.953961588	9224.841495				
16	100	14	4.043923111	9207.568692				
17	85	15	4.136624371	6538.885518				
18	75	18	4.432013724	4979.840687				
19	70	21	4.755221159	4256.881166				
20	65	24	5.108866421	3586.947881				
21	60	27	5.495815973	2970.706076				
22	50	30	5.919206222	1943.11638				
23	45	33	6.382468945	1491.313705				
24	35	36	6.889359102	790.2081317				
25	25	39	7.443985277	308.213653				
26								

Figure 3. Spreadsheet Format for using Excel's Solver

$G$3 + $G$1*(1-EXP($G$2*A1))

and copy into all of column C, beginning with cell C2. Column D is the square of the difference between the recorded MSWS (Column B) and the predicted MSWS (column C), called *chi squared* (χ^2). The following formula should be entered into cell D2 := (B2-C2) $\wedge$ 2 and copied into all of column D. Cell G4 is the sum of the chi squares values, i.e. $\sum_{i=1}^{n} \chi_i^2$ (i.e. G4 = SUM(D2:D25)).

One particular way of obtaining these optimal parameter values is to *minimize* $\sum_{i=1}^{n} \chi_i^2$ since if the predicted values for the MSWS of Irma are very close to the experimental curve then the value for $\sum_{i=1}^{n} \chi_i^2$ will be small. Squared values are chosen to avoid the cancellation effect since the difference between the computed wind speed and actual wind speed could have either sign. To minimize $\sum_{i=1}^{n} \chi_i^2$, we will use the "Solver" add-in in Excel to find the values of v_b and α that result in the minimum value for $\sum_{i=1}^{n} \chi_i^2$. Note that v_0 does not have to be estimated. Why? The procedure for using Excel's Solver is as follows:

1. You can access Solver in one of two ways, depending on which version of Excel is being used. Under the "Tools" menu select "Solver". A new pop-up window will appear. *Remark*: If you do not see this as an option, the add-in will need to be installed. To access Solver, select "Add-Ins" under the "Tools" menu and check the solver add-in.

   Otherwise, you can access Solver in the Analysis group under the "Data" tab. *Remark*: If you do not see this as an option, the add-in will need to be installed. To access Solver, go to File > Options. Click Add-Ins, and then in the Manage box, select Excel Add-ins. Click go. In the Add-Ins available box, select the Solver Add-in check box, and then click OK. After you load the Solver Add-in, the Solver command is available in the Analysis group on the Data tab.

2. In the box labeled "Set Target Cell", type in $G$4. (This is the sum that we are trying to minimize.)
3. Below this, select "Equal To" the "min" function since we are trying to minimize the value in cell G4.
4. In the box labeled "By Changing Cells" type $G$2:$G$3. This allows the solver to try and vary the values for the limiting wind speed v_b and the decay constant α (which should be in cell G2 and G3) to minimize $\sum_{i=1}^{n} \chi_i^2$.
5. Now click on "Solve". The program will alter your initial values to best fit the data.
6. A new pop-up window will appear asking if you want to keep the new values or revert to your original values. Select "keep solver solution" and click the "OK" button. This gives the optimal values for v_0 and α.

Problem 5

If you plotted the solution to the ODE obtained in Problem 3 with the optimal values of A obtained in Problem 4, you should get a graph that resembles Figure 4.

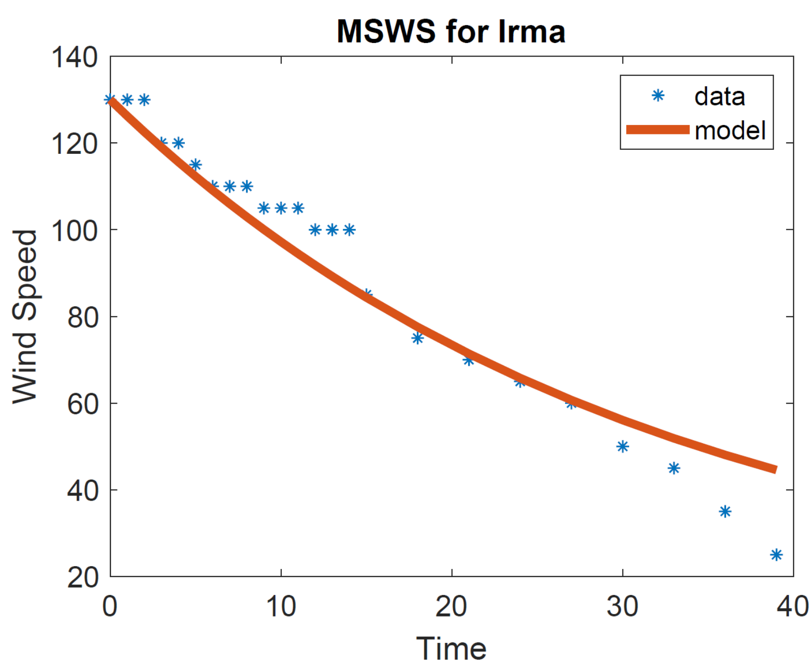

Figure 4. Predicted MSWS vs. Actual MSWS for Irma

Using your graph as a guide, explain how your results verify the following observed claim regarding tropical storms:

Tropical storms whose circulations are partially over water (close to the coastline) decay less rapidly than those that are entirely over land.

Why do you think this is the case? How could you adjust your model to account for this observation?

Problem 6

During the 2018 hurricane season, Hurricane Michael roared ashore near the Florida Panhandle with a maximum sustained wind speed of 155 mph. Hurricane Michael formed in the Gulf of Mexico and made its way northwards until it made landfall near Panama City, FL. This made Hurricane Michael one of the strongest storms to ever hit the United States–and the first Category 5 storm to hit the United States since Hurricane Andrew in 1992. See Figure 5 for Hurricane Michael's track. Consider the data in Table 2, obtained from [4], which gives the MSWS for Hurricane Michael:

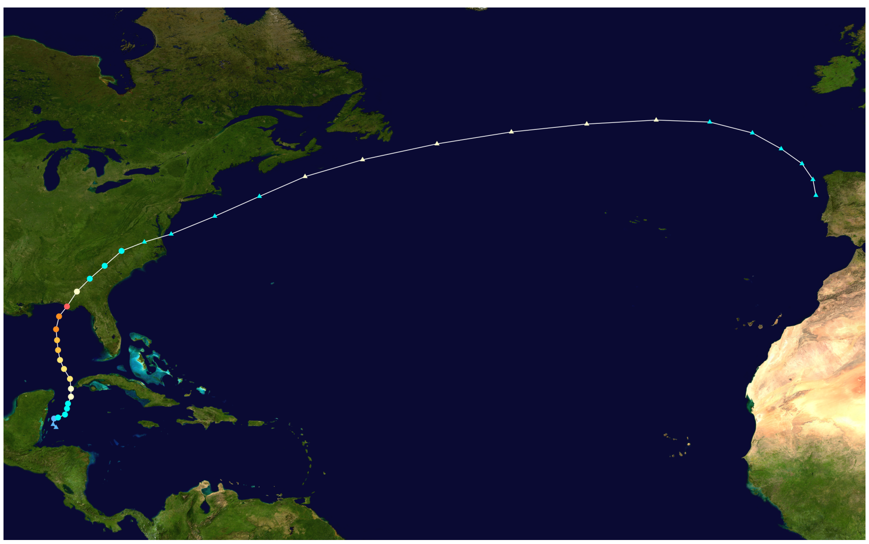

Figure 5. The track of Hurricane Michael [5]

Using the previous problems as a guide, develop a mathematical model which determines the maximum sustained wind speed (MSWS) for Hurricane Michael as a function of time. Construct a plot similar to Figure 4. Discuss your results. Is there an issue of "overpredicting" (predicted wind speeds higher than the actual wind speed) and/or "underpredicting" (as observed with the Irma model)? What possible adjustments could be made to this model to increase its accuracy?

REFERENCES

[1] John Kaplan and Mark DeMaria. 1995. A Simple Empirical Model for Predicting the Decay of Tropical Cyclone Winds after Landfall. *Journal of Applied Meteorology*. 34: 2499-2512.

time after landfall (in hours)	**MSWS (in mph)**	**time after landfall (in hours)**	**MSWS (in mph)**
0	160	10.5	70
0.5	155	13.5	60
1.5	150	16.5	50
2.5	140	19.5	50
3.5	125	22.5	50
4.5	115	25.5	50
5.5	100	28.5	50
6.5	90	31.5	50
7.5	85	34.5	50
8.5	80	37.5	60
9.5	75	40.5	65

Table 2. Michael's MSWS after making landfall in Florida. [4]

[2] Emanuel, Kerry. 2005. *Divine Wind: The History and Science of Hurricanes*. New York: Oxford University Press.

[3] Maximum Sustained Wind Speeds for Hurricane Irma. *Weather Underground* TWC Product and Technology, `https://www.wunderground.com/hurricane/atlantic/2017/hurricane-irma?map=history`. Accessed 31 March 2019

[4] Maximum Sustained Wind Speeds for Hurricane Michael. *Weather Underground* TWC Product and Technology, `https://www.wunderground.com/hurricane/atlantic/2018/Post-Tropical-Cyclone-Michael`. Accessed 18 July 2019

[5] John L. Beven II, Robbie Berg, and Andrew Hagan "Tropical Cyclone Report: Hurricane Michael." *National Hurricane Center* NOAA, `https://www.nhc.noaa.gov/data/tcr/AL142018_Michael.pdf`. Accessed 18 July 2019.

[6] John P. Cangialosi, Andrew S. Latto, Robbie J. Berg. "Hurricane Irma (AL112017) Tropical Cyclone Report." *National Hurricane Center*. NOAA. `https://www.nhc.noaa.gov/data/tcr/AL112017_Irma.pdf`. Accessed 18 July 2019.