1-101-T-Mma-ClassDeathImmigration-TeacherVersion

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA
BrianWinkel@simiode.org

We model the data from Case (E) found in the Excel file 1-101-S-Excel-DeathImmigrationS.xls.

CASE : E - m (0) = 30

This is the equation we believe appropriate for difference equation model. Here is the Procedure and Rule.

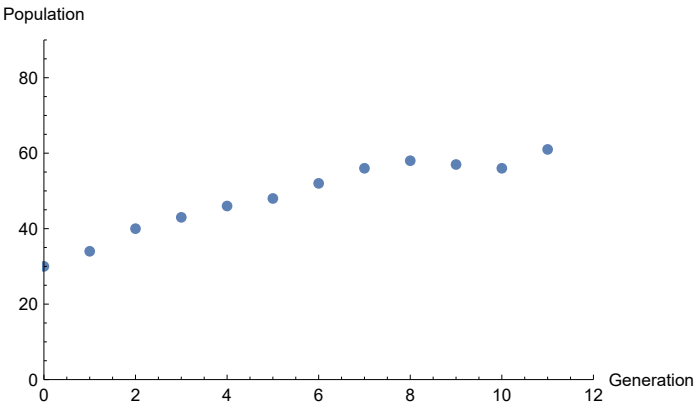
PROCEDURE: Toss living m&m's at generation n and if "m" faces down that m&m lives to next generation.If "m" faces up that m&m is susceptible for deathUpon determining number who survive at generation n then immigrate b = 7 into the population.

RULE (for death) : If die is 6, then the m&m with "m" up really does die.

Here is the data we actually generated using the m&m's.

```
data = {{0, 30}, {1, 34}, {2, 40}, {3, 43}, {4, 46},
   {5, 48}, {6, 52}, {7, 56}, {8, 58}, {9, 57}, {10, 56}, {11, 61}};
```

```
dataPlot =
 ListPlot[data, PlotRange → {{0, 12}, {0, 90}}, AxesLabel → {"Generation", "Population"}]
```



This is the difference equation for the general model and its solution.

```
modSol[n_, a_, b_] =
 m[n] /. RSolve[{m[n + 1] == a m[n] + + b, m[0] == 30}, m[n], n][[1]] // Simplify
```

$$\frac{30\,a^{1+n} + a^{n}\,(-30 + b) - b}{-1 + a}$$

We now form the su9m of square errors between data and model at each generation.

```
SSE[a_, b_] = Sum[(modSol[i, a, b] - data[[i, 2]])^2, {i, 1, Length[data]}]
```

$$\left(-30+\frac{30\,a^2+a\,(-30+b)-b}{-1+a}\right)^2+\left(-34+\frac{30\,a^3+a^2\,(-30+b)-b}{-1+a}\right)^2+\left(-40+\frac{30\,a^4+a^3\,(-30+b)-b}{-1+a}\right)^2+\left(-43+\frac{30\,a^5+a^4\,(-30+b)-b}{-1+a}\right)^2+\left(-46+\frac{30\,a^6+a^5\,(-30+b)-b}{-1+a}\right)^2+\left(-48+\frac{30\,a^7+a^6\,(-30+b)-b}{-1+a}\right)^2+\left(-52+\frac{30\,a^8+a^7\,(-30+b)-b}{-1+a}\right)^2+\left(-56+\frac{30\,a^9+a^8\,(-30+b)-b}{-1+a}\right)^2+\left(-58+\frac{30\,a^{10}+a^9\,(-30+b)-b}{-1+a}\right)^2+\left(-57+\frac{30\,a^{11}+a^{10}\,(-30+b)-b}{-1+a}\right)^2+\left(-56+\frac{30\,a^{12}+a^{11}\,(-30+b)-b}{-1+a}\right)^2+\left(-61+\frac{30\,a^{13}+a^{12}\,(-30+b)-b}{-1+a}\right)^2$$

We find the parameters a and b which minimize the sum of square errors.

```
solab = FindMinimum[SSE[a, b], {a, .8}, {b,
   5}]
```

```
{48.6919, {a → 0.937353, b → 5.48219}}
```

```
asol = a /. solab[[2]]
```

```
0.937353
```

```
bsol = b /. solab[[2]]
```

```
5.48219
```

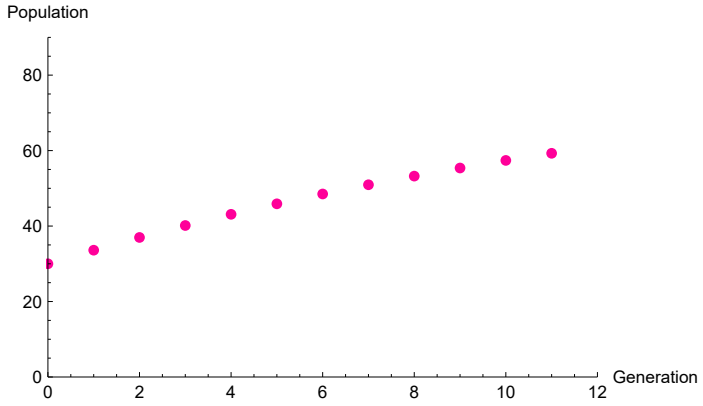
Here is our model with the best parameter estimates used.

```
Model[n_] = modSol[n, a, b] /. solab[[2]] // Simplify
```

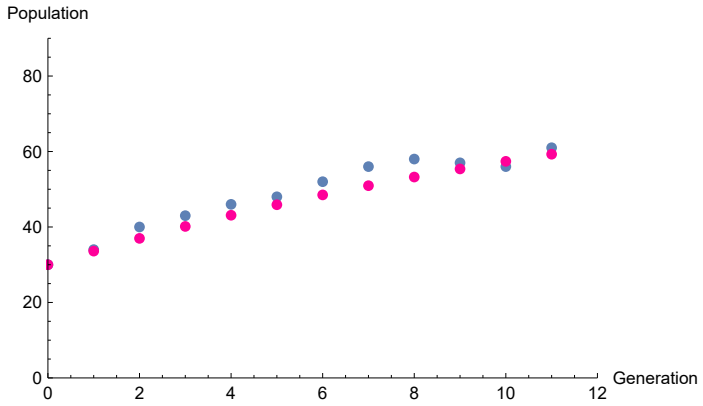
```
87.509 - 57.509 × 0.937353^n
```

We plot our model and then plot it in comparison with the actual data.

```
modelPlot = ListPlot[Table[{n, Model[n]}, {n, 0, 11}], PlotRange → {{0, 12}, {0, 90}},
  PlotStyle → Hue[.9], AxesLabel → {"Generation", "Population"}]
```




```
Show[dataPlot, modelPlot, AxesLabel → {"Generation", "Population"}]
```



We note that the limiting value of this population run is about 87.5.

```
Limit[Model[n], n → Infinity]
```

```
87.509
```

If we are to try to estimate the RULE, i.e., k in following equation, by which we determine which tossed with “m” up m&m lives in the model

$$m[n+1] = (.5 + .5\, k/6)\, m[n] + b$$

we need to solve the following equation given our parameter we estimated as asol = 5.48219, we obtain that k = 5.24 or rounded to nearest Integer, k = 5, so our set of die values must be 6 - 5 = 1 in length, which it was actually.

```
Solve[.5 + .5 k / 6 == asol, k]
```

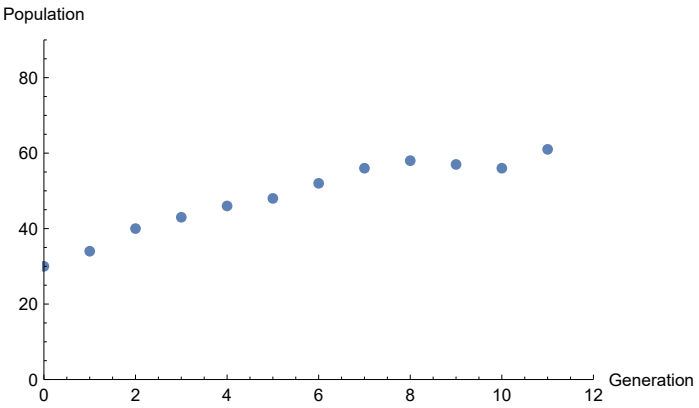
```
{{k → 5.24823}}
```

This is the differential equation for the general model and its solution. Taken

from the COMMENTS section of the document, 1-101-T-ClassM&MDeathImmigration.pdf.

```
data = {{0, 30}, {1, 34}, {2, 40}, {3, 43}, {4, 46},
   {5, 48}, {6, 52}, {7, 56}, {8, 58}, {9, 57}, {10, 56}, {11, 61}};
```

```
dataPlot =
 ListPlot[data, PlotRange → {{0, 12}, {0, 90}}, AxesLabel → {"Generation", "Population"}]
```




```
modSolD[t_, p_, b_] =
 m[t] /. DSolve[{m'[t] == (-.5 + .5 p) m[t] + b, m[0] == 30}, m[t], t][[1]] // Simplify
```

$$\frac{b\left(-2.+2.\,e^{0.5\,(-1.+p)\,t}\right)+e^{0.5\,(-1.+p)\,t}\,(-30.+30.\,p)}{-1.+p}$$

We now form the sum of square errors between data and model at each generation.

```
SSE[p_, b_] = Sum[(modSolD[i, p, b] - data[[i, 2]])^2, {i, 1, Length[data]}]
```

$$\left(-30+\frac{b\left(-2.+2.\,e^{0.5\times(-1.+p)}\right)+e^{0.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-34+\frac{b\left(-2.+2.\,e^{1.\times(-1.+p)}\right)+e^{1.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-40+\frac{b\left(-2.+2.\,e^{1.5\times(-1.+p)}\right)+e^{1.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-43+\frac{b\left(-2.+2.\,e^{2.\times(-1.+p)}\right)+e^{2.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-46+\frac{b\left(-2.+2.\,e^{2.5\times(-1.+p)}\right)+e^{2.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-48+\frac{b\left(-2.+2.\,e^{3.\times(-1.+p)}\right)+e^{3.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-52+\frac{b\left(-2.+2.\,e^{3.5\times(-1.+p)}\right)+e^{3.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-56+\frac{b\left(-2.+2.\,e^{4.\times(-1.+p)}\right)+e^{4.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-58+\frac{b\left(-2.+2.\,e^{4.5\times(-1.+p)}\right)+e^{4.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-57+\frac{b\left(-2.+2.\,e^{5.\times(-1.+p)}\right)+e^{5.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-56+\frac{b\left(-2.+2.\,e^{5.5\times(-1.+p)}\right)+e^{5.5\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2+$$
$$\left(-61+\frac{b\left(-2.+2.\,e^{6.\times(-1.+p)}\right)+e^{6.\times(-1.+p)}\,(-30.+30.\,p)}{-1.+p}\right)^2$$

We find the parameters p and b which minimize the sum of square errors.

```
solab = FindMinimum[SSE[p, b], {p, .8}, {b, 5}]
```

```
{48.6919, {p → 0.870609, b → 5.66144}}
```

```
psol = p /. solab[[2]]
```

```
0.870609
```

```
bsol = b /. solab[[2]]
```

5.66144

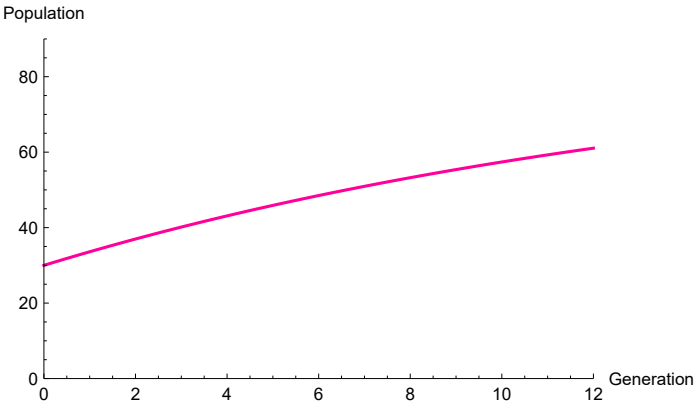
Here is our model with the best parameter estimates used.

```
ModelCont[t_] = modSolD[t, p, b] /. solab[[2]] // Simplify
```

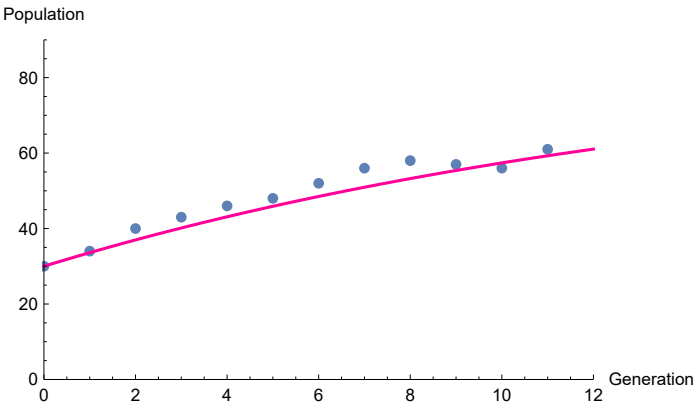
$87.509 - 57.509\, e^{-0.0646954\, t}$

We plot our model and then plot it in comparison with the actual data.

```
modelContPlot = Plot[ModelCont[t], {t, 0, 12}, PlotRange → {{0, 12}, {0, 90}},
  PlotStyle → Hue[.9], AxesLabel → {"Generation", "Population"}]
```




```
Show[dataPlot, modelContPlot, AxesLabel → {"Generation", "Population"}]
```

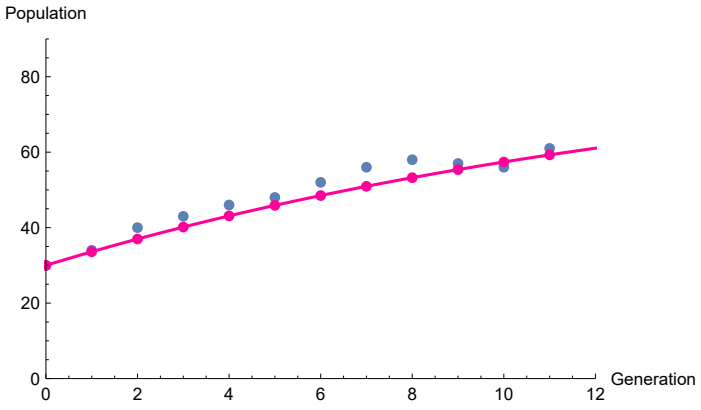


Note that our p = 0.870609 which is quite close to the value in the model, namely p = 5/6 = 0.8333333.

We note that the limiting value of this population run is about 87.5.

```
Limit[ModelCont[t], t → Infinity]
```

87.509

We show the discrete and continuous models (in pink) over the data in blue.

```
Show[dataPlot, modelContPlot, modelPlot, AxesLabel → {"Generation", "Population"}]
```