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STUDENT VERSION
Class M&M Death and Immigration Simulation

Brian Winkel, Director SIMIODE
Chardon OH 44024 USAUSA
BrianWinkel@simiode.org

Abstract: We offer students an opportunity to generate unique data for their team on a death and immigration model using m&m's and then pass on the data to another student team for analysis with a model they built. The key is to recover the parameters and try to explain how the simulation was conducted.

SCENARIO DESCRIPTION

This activity is much like that in [1] except that individual students or teams of students will determine the parameters of their own simulation, run the simulation, and then share the data with other students so the latter students can model and estimate parameters unique to the simulation used to generate the data.

Early Simulation

Each team will generate data from one simulation. Teams will need 1 bag of regular m&m's containing roughly 55 m&m's, two small cup containers (1 Action cup and 1 Holding cup), one paper plate or confining area in which to safely toss m&m's and then collect m&m data, one six sided die, and one sheet of paper for writing down the parameters used and their simulation results.

Each team should count out 20 m&m's and put them in the Action cup, i.e. the one from which we will toss m&m's onto the plate for each generation. If possible try to avoid m&m's which are yellow as it is sometimes hard to detect the "m" on the piece. Put the remaining m&m's in the second cup, a Holding cup.

We are going to toss the m&m's from the Action cup (gently toss!) onto the plate and then softly move the plate around to be sure the m&m's are settled out and laying flat, not braced on one another.

In the simulations of [1] we would note which m&m's had an "m" facing up and these would die. We would remove them from the population and then immigrate 10 m&m's back from the Holding

cup onto the paper plate. Then we would count and note the number of m&m's on the plate and place all these m&m's into the Action cup for our next generational toss. Each toss would be called a *generation*. So generation 0 would have 20 m&m's. We would note the generation and the number of m&m's in our living population by $m(n)$. Then we would toss these m&m's into the plate, noting which m&m's had "m" facing up, kill these off into the Holding cup, and then immigrate 10 m&m's into our population for the next generation.

Thus if we had $m(0) = 20$ and upon tossing these 20 m&m's from our Action cup onto the paper plate we saw 7 m&m's with "m" facing up we would remove 7 m&m's from our plate (leaving 13 m&m's on the plate) into the Holding cup, but then immigrate 10 m&m's from the Holding cup so that our next generation, $m(1)$, would have $m(1) = 23$ m&m's. We would continue in this manner, noting our generational data.

New Simulation

In this new simulation we do everything the same EXCEPT that for each m&m with an "m" facing up we further make a decision about death. Using a six-sided fair die we establish a *RULE* to determine if each m&m with an "m" facing up really does die. For example, for each m&m with an "m" facing up toss the die and if a 3, 4, 5 or 6 faces up THEN the m&m actually does die and can be removed; otherwise this m&m lives. We do this for each potential death for an m&m. We continue to immigrate after death and then note down generation and population size at each generation.

Now you decide your *RULE* before proceeding with your generating data. For example you could be beneficent to the m&m population and use this *RULE*: For each m&m if the die comes up 1 then that m& dies, otherwise the m&m is given a second chance and lives. A second part of your *RULE* will be to determine just how many m&m's (keep this number constant throughout the simulation, though) will immigrate from the Holding cup to your paper plate before recording generational data. Originally we suggested 10 m&m's would immigrate, but you can make that any reasonable constant immigration you wish.

Let us go through the steps of our New Simulation. Keep your *RULE* on a separate sheet from your data sheet and NEVER share that rule with anyone until your instructor asks you to do so.

1. Place 20 m&m's into our Action cup. Note that $m(0) = 20$, i.e. we have 20 m&m's in our population at generation 0. There is nothing sacred about the number of m&m's you use initially except that if you are working with one standard bag of m&m's you probably have a maximum of 55 m&m's to play with, so think about how high your m&m population might go with your rule for death and immigration.
2. Toss the m&m's (gently!) from the Action cup onto the plate and softly move the plate around so the m&s are somewhat separated and clearly facing "up" or "down."
3. For each m&m on the plate with an "m" facing up toss a six-sided die and if a 3, 4, 5 or 6 comes up that m&m actually does die and is removed from the plate and placed in the Holding cup. However, you are to apply your own *RULE* you determined here.

4. After determining deaths then immigrate 10 m&m's from the Holding cup to the paper plate. Be sure to use your *RULE*, though. Again, nothing is sacred about the immigration amount, but realize you are probably going to be limited by the initial size of the m&m supply from one bag, about 55 m&m's in total, so you might not want to have an immigration level of, say 25.
5. Note the generation and the population on a chart. You should have Generation n and Population $m(n)$ for each generation $n = 0, 1, 2, 3, \ldots$. See Table 1.
6. Go to Step (2) and continue until told to stop by your instructor or something obvious happens.

Once each team has completed their simulation they should put their Team Name on their data sheet (should look like Table 1 with Team name at top) and exchange data sheets with other teams in the class so that no team has their own data sheet.

Modeling Death and Immigration of m&m's	
Iteration	# M&Ms at start of iteration
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

Table 1. Modeling death and immigration of m&m's.

Analysis - Activity

Each team should have a set of generational data received from another team and their job is to create a mathematical model for $m(n)$, the number of m&m's alive at generation n.

In effect, each team needs to build a mathematical model which contains a "death" term and an "immigration" term and then estimate these parameters as best they can. Indeed, they should

Generation	# m&m's	# with "m" up	# "m" really die	# m&m's to survive	add 7.
0	20	10	3	17	24
1	24	10	7	17	24
2	24	12	7	17	24
3	24	10	4	20	27
4	27	15	5	22	29
5	29	12	8	21	28
6	28	14	4	24	31
7	31	14	6	25	32
8	32	18	9	23	30
9	30	15	7	23	30
10	30	15	8	22	29
11	29	10	6	23	30
12	30	12	10	20	27
13	27	12	5	22	29
14	29	18	8	21	28
15	28	17	6	22	29
16	29	12	5	24	31

Table 2. The data from simulation (A) .

be able to recover the *RULE* used in each case. Perhaps they cannot recover whether the set of outcomes on the die toss which governs whether an m&m with "m" facing up actually lives was $\{1, 2, 3\}$ or $\{2, 4, 6\}$, but they should be able to estimate the size of this set.

Once the receiving team has estimated the parameters they can then check in with the data generation team to see how well their model is.

If you cannot run this experiment with "equipment" of your own then we offer data we generated in a separate Excel file (one for each of 5 runs (A) - (E) of the simulation in 1-101-S-Excel-DeathImmigrationS.xls).

Additional Activities

You are to model each simulation ((A) - (E)), estimate the parameters involved in your model, and then indicate what you believe the *RULE* was for each (or at least one) run (A) - (E) found in the Excel file, 1-101-S-DeathImmigrationS.xls, as best you can with justification. We enclose in Table 2 the complete generating set of data for simulation (A). All other data sets have the generation data reported only in the spreadsheet.

REFERENCES

[1] Author. 2015. 1-001-S-MandMDeathAndImmigration.