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STUDENT VERSION

Steeping Tea

A differential equations approach to a great cup of fruit tea

Abstract: In this activity, we provide photographs of the steeping process for a fruit tea steeped in hot water. Students build a differential equation model for the steeping process and do parameter estimation using the color of our tea as a way to measure relative concentration of the tea oils in solution. Students then build a differential equation model for the dissolution of a sugar cube in the tea and use data to estimate parameters. Finally, students use Newton's law of cooling to estimate the temperature of our tea after it has steeped for a certain amount of time and after the sugar has been completely dissolved.

SCENARIO DESCRIPTION

There is just nothing better than a great cup of tea! In this activity, you will be given experimental data for the steeping of a fruit tea, the dissolution of a sugar cube in the tea, and the changing temperature of the tea. Your job will be to model each of these three processes using differential equations and present your findings in a technical report.

Your Tasks:

Please view all the tasks, data sets, images, and hints before you start the tasks in order to understand the full scope of the problem.

1. Use the color and/or gray scale data given in Table 1 (and shown pictorially in Figure 1) to develop, analyze, and solve a differential equation model for the concentration of the tea. In this case, color is being used as a proxy measure for the concentration of the dissolved tea oils. In Table 1 we give the red, green, blue (RGB), and grayscale color values. The grayscale values were calculated from the RGB values using the formula

$$x = 0.288R + 0.587G + 0.114B \tag{1}$$

where x is the grayscale value corresponding to the RGB value of a pixel [1]. The RGB values were extracted from the photographs using Adobe Photoshop. A sample of 10 pixels were

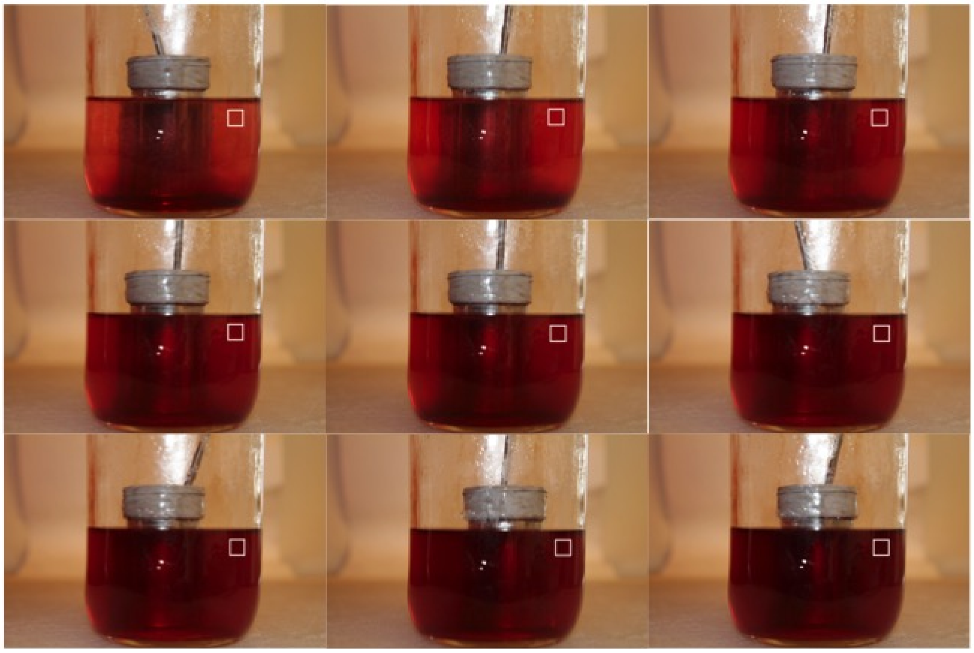

Figure 1. The time evolution of the tea steeping. The top left image is at $t = 60$ seconds; then reading along the rows for $t = 90$ seconds and $t = 120$ second on the first row. The second row of images shows $t = 150$ seconds, $t = 180$ seconds, and $t = 210$ seconds. The third row has $t = 240$ seconds, $t = 270$ seconds (left and middle) and $t = 600$ seconds (bottom right). The white squares indicate where the pixel samples was taken in the photo software.

chosen in the white squares indicated in Figure 1 and the averaged and rounded RGB values are given in Table 1.

Table 2 gives the meanings of the color values. The color values are common across photography software. Notice in particular that a photo that is pure white has a grayscale value of 255 and a photo that is pure black has a grayscale value of 0. Since the tea is changing color from lighter to darker we expect the grayscale value to be decreasing with time. It should be noted that the extraction of RGB color values can be done with any free or proprietary photo editing software (e.g. Photoshop, MS Paint, Digital Color Meter (Mac), etc).

2. Use the small set of sugar cube data in Table 3 to develop, analyze, and solve a differential equation model for the dissolution of a 1cm^3 sugar cube in the tea. Determine the amount of time that it takes to dissolve the sugar cube. (Hint: it completely dissolves). Include a plot of the time evolution of the sugar dissolution. Be sure that your data points are visible on the plot.

Time (seconds)	Temperature (°F)	R color value	G color value	B color value	Grayscale
≈ 5	160	190	138	80	144.846
60	155	170	45	15	77.085
90		161	20	10	59.248
120		136	13	5	47.369
150		131	13	5	40.701
180	145	121	9	9	46.385
210	142	108	9	4	36.843
240		96	7	1	31.871
270		90	4	3	28.61
600	120	73	5	2	24.187

Table 1. Temperature and color data. The color data for time $t \approx 5$ was taken from Figure 2 and the remainder of the color data was taken from Figure 1. The Grayscale conversion was calculated via $x = 0.288R + 0.587G + 0.114B$ as found in [1].

Color	R color value	G color value	B color value	Grayscale value
Pure black	0	0	0	0
Pure white	255	255	255	255
Pure red	255	0	0	-
Pure green	0	255	0	-
Pure blue	0	0	255	-

Table 2. The meaning of the RGB color values pulled from the photo software.

Time (seconds)	Side Length (cm)
0	1
30	0.6

Table 3. The sugar cube data. Due to the quick dissolution of the sugar cube we only have two data points.

3. Use the temperature data in Table 1 to develop, analyze, and solve a differential equation model for the temperature of the tea during the time that it takes to steep. Include a plot of the time evolution of the temperature. Be sure that your data points are visible on the plot.
4. Write a clear paper to a technically sophisticated audience giving the details of your differential equation modeling tasks and your results. Be sure to state and defend all of your assumptions in clear language.

Experimental Setup:

Now we give the details of the experimental setup.

Equipment: The following equipment was used to gather the data.

Container: Standard large Mason jar for canning filled with 1.5 cups of hot water. See Figure 2.

Figure 2. The experimental equipment. The image on the left shows the setup at roughly $t \approx 5$ seconds. The right image shows the porous cylindrical tea steeper, the thermometer, the tea, and the Mason jar.

Tea: *Steeped Tea* brand Jenna Cherry Jubilee Fruit Tea. Suggested instructions are for 4 to 5 minutes of steeping in hot water with 1.5 to 2 teaspoons of tea. In this experiment we used 2 teaspoons of tea in a porous metal tea steeper (not in a standard teabag). According the packaging the ingredients for the tea were: Hibiscus petals, rose hip, apple pieces, banana pieces, cherry pieces, strawberry pieces, orange pieces, and natural flavors.

Steeper: Metal porous cylindrical tea steeper. The cylinder has a height of 2 inches measured from the bottom of the container to the top of the pores. The cylinder has a constant diameter of 1.5 inches.

Thermometer: Standard (clean) kitchen meat thermometer.

Camera: Canon EOS 60D

Data Collection: Data was collected on Jan 21, 2017 starting at 10AM MST.

- The ambient temperature in the room for the duration of the experiment was approximately constant at 66°F.

- The water was initially warmed to 160°F and then poured into the Mason jar. The tea steeper, containing 2 teaspoons of the tea, was placed into the water so that the water level was over top of the highest pores. Temperature was then gathered periodically throughout the experiment. The left side of Figure 2 shows an image of the Mason jar and tea steeper at time $t \approx 5$ seconds and the right side of Figure 2 shows the porous tea steeper, the tea packaging, the thermometer, and the Mason jar. See Table 1 for temperature data.
- The Canon EOS 60D camera was set to take a picture every 30 seconds starting at the 1 minute mark. The camera was placed on a tripod and was not touched during the process.
- Pictures were taken until 4 minutes and 30 seconds and then the tea was allowed to cool and continue to steep until the 10 minute mark, at which point one more picture and temperature reading were taken. The 10 minute data for color was taken to represent a maximum color saturation.
- Lighting was held approximately constant during the 10 minute experiment. See Table 1 for the RGB color value data and see Table 2 for the meaning of the RGB color values. The color values were converted to grayscale using the conversion found in [1].
- The tea was stirred with 6 counterclockwise rotations around the steeping cylinder every 30 seconds starting from $t \approx 5$ seconds to just before the 10 minute mark.
- See Figure 1 for the time evolution starting at 1 minute reading across the rows until 4 minutes 30 seconds. The bottom right image in Figure 1 shows the tea at 10 minutes. Adobe Photoshop was used to extract RGB pixel values for each image. Pixel values given are the rounded average of 10 randomly selected pixels from inside the white squares in Figure 1.
- The sugar cube was dissolved in the Mason jar starting at a temperature of 150°F and stirred slowly until it was invisible. See the sugar cube data in Table 3. A standard set of calipers was used to measure the side length of the sugar cube.

Suggestions

1. You do not need to do the tasks in order. Some of the tasks should be easier than others.
2. For each task try to initially come up with the simplest differential equation model. Make several simplifying assumptions, if necessary, but be sure to state them clearly in your write-up. The simplest differential equations for this scenario are likely first order, but they are not necessarily linear.
3. You are allowed to use any appropriate software and any by-hand solution techniques for the differential equations. Software is likely not necessary for most parts of this activity so don't get stuck by going to the computer first.
4. You will need to do some parameter estimation for this problem and parameter estimation is usually best done with software. There are several software tools available for doing this and

in the list below we give you a few videos with step-by-step instructions for how to use the software. The instructions are given on a data set unrelated to this problem so you will need to modify anything you see in the videos to work for these problems.

- Parameter estimation in Excel: `https://youtu.be/mEthE6Hia-k`
- Parameter estimation in MATLAB: `https://youtu.be/9mmm5N0lz5o`
- Parameter estimation in R: `https://youtu.be/9qjIrEyQiAk`

5. For the concentration problem you may want to start by modeling the process in which the tea oil is leached from the tea leaves and then conjecture a simple mathematical formula to convert to the grayscale color value. Be sure to be clear when you write up your process.
6. Be creative and have fun!

REFERENCES

[1] Wikipedia article for grayscale.https://en.m.wikipedia.org/wiki/Grayscale. Accessed January 2017.