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STUDENT VERSION
EMPTY SPHERICAL TANK

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA USA
BrianWinkel@SIMIODE.org

Abstract: We model the emptying of water from a spherical tank. First, we pump out water at a constant rate. Second, we allow the water to exit through a small hole in the bottom of the tank. We seek to determine how fast the water level is falling in both cases. Furthermore, we seek to determine the size of the small hole at the bottom so that the time it takes for all the water to run out of the spherical tank is the same as the time it takes to pump the water out at a constant rate.

SCENARIO DESCRIPTION

There are several ways to empty a container filled with water, among them are (1) through a pump at a constant rate and (2) through a hole in the bottom of the container.

We consider a spherical tank in which we first pump the water out at a constant rate and then permit the water to fall through a hole in the bottom.

We wish to model both and determine the size of the hole in the bottom of the tank which will give us the same time for emptying the tank as when we pump out the water at a constant rate.

In both cases we seek to model the situation so that we can predict the height of the water level at time t.

Let us consider a thin-shelled spherical tank if inner radius R m. Suppose the tank is completely filled with water. We will use the numerical value of $R = 2$ m and for the constant pump rate we will use $B = -0.1\text{m}^3/\text{s}$.

Torricelli's Law for the height of a column of water, $h(t)$, at time t in a container of cross sectional area $A(h(t))$ when the water is at height $h(t)$ says $h(t)$ must satisfy the differential equation (1),

$$A(h(t)) \cdot h'(t) = -a\alpha\sqrt{2gh(t)}\,. \tag{1}$$

g is the acceleration due to gravity, a is the cross sectional area of the hole through which the water exits from the tank, and α is an empirical number which indicates the percentage of the maximum

flow rate which actually gets through the small hole, the reduction due to friction and constriction. α is called the *discharge or contraction coefficient*. Note the negative sign on the right hand side of (1) as water is leaving the column of water. You can see the development of Torricelli's Law in the Modeling Scenario [1].

We assume the small hole at the bottom of the spherical tank is circular of radius r, i.e. $a = \pi r^2$.

The issue is then to decide how big a hole, i.e. determine r in $a = \pi r^2$, so that the time it takes the water to empty from the tank in this case is the same as the time it takes when the water is pumped out. Go ahead and determine just how big the hole has to be for this to happen and compare the volumes, height, and rate of change in volume for both cases.

REFERENCES

[1] Winkel. B. 2015. 1-015-S-Torricelli.