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STUDENT VERSION
Bacteria Growth

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Abstract: We offer students a simulation experience or data from a simulation and ask them to model the simulation using several approaches, to include exponential growth fit, difference equation, differential equation, and parameter estimation using EXCEL spreadsheet. In this particular modeling scenario, we know the exact solution and want to see how various models predict our expectations. We have used EXCEL and *WolframAlpha* for students to experience and appreciate modeling approaches without worrying much about technology and programming.

SCENARIO DESCRIPTION

Bacteria Growth:

Imagine that we observe bacteria that undergo cell division and divide every twenty minutes to reproduce exactly two copies of themselves. At the start of the experiment assume that there was exactly one bacterium. We would like to ask, how will the number of bacteria change over time? We have sketched a diagram in Figure 1 to have a better understanding of the growth of bacteria in discrete time. In order to describe our observation, we call the initial time, the time when we started the experiment is equal to zero where the number of bacterium is one. After twenty minutes, the bacterium subdivides into two. Twenty minutes later, each bacterium splits further, resulting in four bacteria at time 40 minutes, and so on.

Collect the data:

In order to describe our data set of two columns first with time in minutes and second with number of bacteria. Table 1 shows the data set for bacteria growth with discrete time. We can simplify the data if we measure time in a more convenient way. Considering one unit of time to be equal to 20 minutes. The data is visualized in Figure 2.

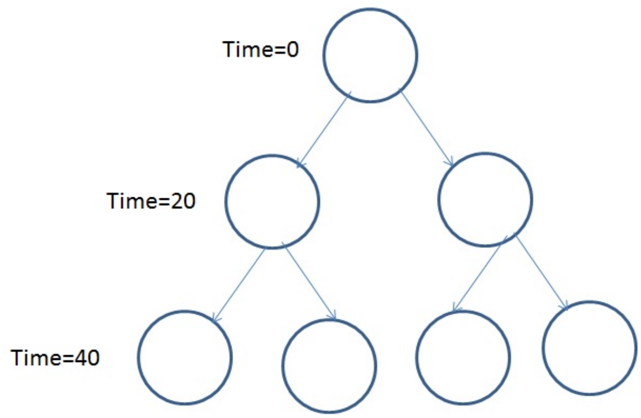

Figure 1. Bacteria split every 20 minutes.

(a) Build a model for $a(n)$, the population growth of bacteria after n units of time, where $a(0) = 1$. State your assumptions clearly and show how they are used in the model building process. Compare your model prediction with the data.

(b) Build a model with a difference equation for $a(n)$ the population growth of bacteria after n units of time, where $a(0) = 1$. State your assumptions clearly and show how they are used in the model building process. Compare your model prediction with the data.

(c) Build a model with a differential equation for $a(n)$ as a continuous population growth of bacteria after n units of time, where $a(0) = 1$. Compare your model prediction with the data.

(d) Build a model for parameter estimation for the differential equation model where $a(n)$ represents the continuous population growth of bacteria after n units of time, and $a(0) = 1$. Compare your model prediction with the data and estimate the parameter value and error of estimates.

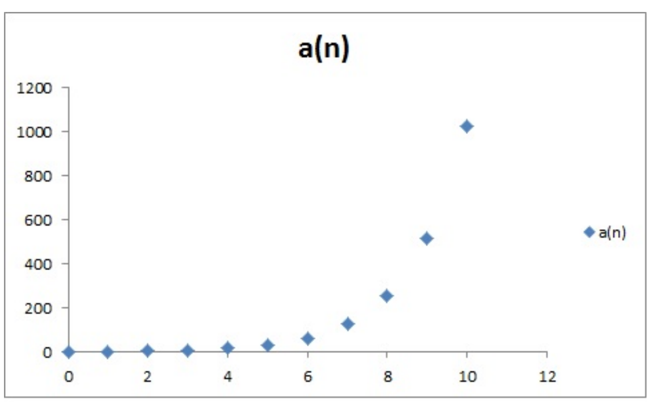

Figure 2. Plot of the data from Table 1 in which n is the time unit and $a(n)$ is the number of bacteria growth.

Time(min)	Time Unit	Number of Bacteria
	n	$a(n)$
0	0	1
20	1	2
40	2	4
60	3	8
80	4	16
100	5	32
120	6	64
140	7	128
160	8	256
180	9	512
200	10	1024

Table 1. This simulation describes the growth of bacteria population in every twenty minutes where one unit of time is considered as 20 minutes.