www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

TEACHER VERSION
SALT WATER TANK MODELING

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA
BrianWinkel@SIMIODE.org

Abstract: We offer three mixing problems, of increasing order of difficulty, in which salt is coming into a tank of water and upon instantaneous mixing is leaving the tank.

SCENARIO DESCRIPTION

Consider the following situations and address the issues raised.

SITUATION 1: At time $t = 0$ a tank contains Q_0 lb of salt dissolved in 100 gal of water. Assume that water containing $\frac{1}{4}$ lb of salt per gallon is entering the tank at a rate of 3 gal/mm, and that the well-stirred solution is leaving the tank at the same rate.

(a) Describe what you think will happen to the amount of salt in the tank over time, perhaps with a verbal description or a plot.

(b) Describe the rate at which the salt water is being added to the tank and "subtracted" from the tank as a function of time t in s.

(c) Write out a differential equation with initial condition which describes the rate of change of salt in the tank, i.e. $Q'(t)$.

(d) Find an expression for the amount of salt $Q(t)$ in the tank at time$t$.

(e) Describe the long term behavior of the amount of salt in the tank. Tell why this makes sense.

(f) Determine the maximum or maximum possible amount of salt in the tank over time.

(g) Determine the time when the amount of salt in the tank is at 75% of the maximum possible in (e).

Let us consider a variation on SITUATION 1:

SITUATION 2: At time $t = 0$ a tank contains Q_0 lb of salt dissolved in 100 gal of water. Assume that water containing $\frac{1}{4}$ lb of salt per gallon is entering the tank at a rate of 3 gal/mm, and that the well-stirred solution is leaving the tank at the rate of 3.5 gal/min. Let $Q(t)$ be the number of pounds of salt in the tank at time t min.

(a) Describe what you think will happen to the amount of salt in the tank over time, perhaps with a verbal description or a plot.
(b) Describe the rate at which the salt water is being added to the tank and "subtracted" from the tank as a function of time t in min.
(c) Write out a differential equation with initial condition which describes the rate of change of salt in the tank, i.e. $Q'(t)$.
(d) Find an expression for the amount of salt $Q(t)$ in the tank at time$t$.
(e) Describe the long term behavior of the amount of salt in the tank. Tell why this makes sense.
(f) Determine the maximum or maximum possible amount of salt in the tank over time.
(g) Determine the time when the amount of salt in the tank is at 75% of the maximum possible in (e).

Finally, let us consider a scenario which is quite different:

SITUATION 3: A 1200 gal tank has 400 gal of water in it containing exactly 20 lb of salt at the start of the process. Salt water is being added to the tank. The concentration of the salt in the water being added is 0.03 lb/gal. This salt water mixture initially is added at a rate of 0.1 gal/s and this rate is steadily increased by 0.02 gal/s each minute. The water in the tank is presumed to mix instantaneously and this mixture is being pumped out of the tank at the rate of 5 gal/s.

Let $S(t)$ be the amount of salt in the tank at time t (s).

(a) Describe what you think will happen to the amount of salt in the tank over time, perhaps with a verbal description or a plot.
(b) Describe the rate at which the salt water is being added to the tank as a function of time t in min.
(c) Write out a differential equation with initial condition which describes the rate of change of salt in the tank, i.e. $S'(t)$.
(d) Determine the amount of salt in the tank at time t for the first 500 s of operation.
(e) Plot the amount of salt in the tank as a function of time$t$.
(f) What is the minimum amount of salt in the tank during the first 500 s of operation?

NOTES FOR TEACHER

We believe our analyses offered in a Mathematica notebook 1-52-T-Mma-SaltWaterTank.nb or the pdf version for those who do not read Mathematica files, which is found in the Supplementary Docs material, demonstrates the basic approach a student should aspire to in a submission write-up.

This Modeling Scenario is an example of formulating an existing paper from the literature [2], in this case one from the author published over 20 years ago. Moreover, it is an illustration of how contributors can mine the literature, perhaps created by them or created by others. For the latter see Modeling Scenario, 3-19-T-ShuttleCock [3] in which data an one approach for parameter estimation is featured from [1].

REFERENCES

[1] Peastrel, M., R.Lynch, and A. Armenti, Jr. 1980. Terminal velocity of a shuttlecock in vertical fall. *American Journal of Physics*. 48(7): 511-513.

[2] Winkel, B. J. 1994. Modelling mixing problems with differential equations gives rise to interesting questions. *International Journal of Mathematical Education in Science and Technology*. 25(1): 55-60.

[3] Winkel, B. J. 2015. 3-019-S-ShuttlecockFall.