www.simiode.org SINIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION PROPAGATION OF THE WORD JUMBO

Rachel L. BaylessRachelle C. DeCosteAgnes Scott CollegeWheaton CollegeDecatur GA 30030USANorton MA02766 USArbayless@agnesscott.edudecoste_rachelle@wheatoncollege.edu

Abstract: This activity is a gentle introduction to modeling via differential equations. The students will learn about exponential growth by modeling the rate at which the word jumbo has propagated through English language texts over time.

SCENARIO DESCRIPTION

In 1861, the elephant who became known as Jumbo was captured in Ethiopia and purchased by animal collector Johan Schmidt. First known as the "Children's Giant Pet," the largest elephant known at the time was renamed Jumbo, a word believed to have been created from the combination of the Swahili words *jumbe* and *jambo*, meaning "chief" and "hello", respectively. [1]

Eventually Jumbo was traded to the London Royal Zoological Society. By age seven the 11.6 foot, 6.5 ton bull's daily intake included "200 pounds of hay, 1 barrel of potatoes, 2 bushels of oats, a slew of onions and several pails of water." To address health concerns, his zookeeper Matthew Scott gave Jumbo a gallon of whiskey per day when he felt it was necessary. [1]

Despite much protest from English residents, in 1882 Jumbo was acquired by P.T. Barnum to be added to his circus, "The Greatest Show on Earth." For three years, until his death in 1885, Jumbo traveled with the circus on a rail car that was specifically made to be large enough to hold the elephant. After his death, Jumbo's hide was stuffed and continued to tour with the circus until it was donated to Tufts University for display in the Barnum Museum of Natural History. Tufts' athletic teams were named after the elephant, and his hide was seen as a source of good luck. Jumbo's bones were mounted and displayed in the American Museum of Natural History in New York. [2]

Today the word jumbo is commonly understood to mean very large, as in jumbo jet and jumbo shrimp. Through this project we will explore how the word jumbo has spread through the English language based on its original use in the 19th century.

In your Calculus II class, you may have been briefly introduced to differential equations; some differential equations can be solved with integration. One common model studied is population growth. The most basic model assumes that population growth should be proportional to the size of a population. This leads to the following differential equation:

$$\frac{dP}{dt} = kP$$

where P(t) is the population at time t and k is the constant that is determined by how quickly the population grows. You may want to keep this example in mind as you consider the following questions with your group.

- 1. How do you quantify a word's popularity?
- 2. Suppose you have a sentence consisting of 10 words, and the word jumbo appears exactly once. On the other hand, suppose you have a paragraph consisting of 100 words, and the word jumbo appears exactly once. Does this example change your definition of popularity?
- 3. We want popularity to be a function of time. Let us define t to be years. The word jumbo first appeared in print in 1861; thus our function will only be defined for $t \ge 1861$. Then define popularity as a function of time t by P(t) = Percentage of words published in books in year t that is the word jumbo. Explain why, specifically,

$$P(t) = \frac{\text{\# of instances of jumbo in books published in year } t}{\text{\# of words published in books in year } t} \times 100$$

provides a reasonable measure of popularity.

4. Explain (in words) what $P(1900) = 2.08 \times 10^{-6}$ means.

5. Explain (in words) what $\frac{dP}{dt}$ represents.

- 6. Explain (in words) what factors or variables could influence $\frac{dP}{dt}$ and how.
- 7. Consider the scatter plot (Figure 7) of data from Google Ngrams [3] along with your answer to the question above. Given this new information, what factor appears to be influencing $\frac{dP}{dt}$?



Figure 1. A scatter plot of percentage of printed words that are the word jumbo versus time for the years 1880 to 1940.

8. Write a mathematical expression for $\frac{dP}{dt}$.

Consider the following differential equation,

$$\frac{dP}{dt} = kP,\tag{1}$$

where P(t) is the percentage of all words published in books in year t that is the word jumbo.

- 9. Explain (in words) what it means if k = .05.
- 10. Explain (in words) what it means if k < 0.
- 11. Solve the differential equation in (1). That is, find a function P(t) such that its derivative is a constant multiple of the function itself. (There is more than one way to do this. Explain your thinking.)
- 12. Returning to thinking of P(t) as the percentage of total words in books that is the word jumbo, draw the general shape of the graph.
- 13. What does the model forecast will happen as t increases? Does the model continue to be valid? Explain.

Year	Total Words	Jumbo Appearances	Percentage
1880	784223075	6	$7.65088428442379 \times 10^{-9}$
1881	789254798	12	$1.52042154579338 \times 10^{-8}$
1882	828502461	1	$11.20699701820198 \times 10^{-9}$
1883	930196929	17	$1.82756999835247 \times 10^{-8}$
1884	881638914	3	$3.40275361302847 \times 10^{-9}$

Table 1. The first five rows of a spreadsheet with the dataon yearly usage of the word jumbo in published books.



Figure 2. A scatter plot of percentage of printed words that are the word jumbo versus time for the years 1880 to 1940. The red line is the best-fit exponential, $P(t) = 5.07231223e^{.0549975t}$.

We would now like to check that our theoretical model fits the real data from Google Ngrams. First, Table 1 is an excerpt from the raw data [3]. It shows the year, total words published, the number of times jumbo appeared, and the percentage of the published words that are the word jumbo.

From these data we can create a scatter plot of percentage of printed words that are the word jumbo versus time which is overlaid with an appropriate exponential.

Acknowledgments

We wish to thank Professor Michael Drout of Wheaton College for suggesting a project using Google Ngrams. We also appreciate Giampaolo Di Gregorio's suggestion of historical references for the etymology of the word jumbo. The authors also thank the referees for their thoughtful and helpful feedback on improving this project.

REFERENCES

- Jumbo the Elephant. The Barnum Museum. https://www.barnummuseumexhibitions.org/jumbotheelephant.htm. Accessed 15 July 2015.
- [2] Sauer, Anne. The Story of Jumbo. Originally found in Tufts Journal, October, 2001. Reprinted online at http://tuftsjournal.tufts.edu/archive/2001/october/tufts150/index.shtml. Accessed 15 July 2015.
- [3] Google Books Ngram Viewer. http://books.google.com/ngrams. Accessed 15 July 2015.