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STUDENT VERSION
SLEUTHING WITH DIFFERENTIAL EQUATIONS

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Abstract: We present several situations in which differential equation models serve to aid in sleuthing and general investigations. One involves initial speed given information about constant deceleration and distance to stop in traffic incident; one involves modeling a steel ball launched vertically and some issues about flight; and two involve time of death issues using Newton's Law of Cooling and Sherlock Holmes' quick analysis.

SCENARIO DESCRIPTION

Contested speeding violation and the wheels of justice

In a 35 miles per hour speed zone a car driver applies and holds the brakes to the max and thus decelerates the car at 12 ft/s^2 (according to the manufacturer specifications and settings) until the car stops in 225 ft.

a) How fast was the car traveling at the instant the driver applied the brakes? Make a recommendation to the law enforcement agency. Defend your recommendation with analysis.
b) The driver hires a lawyer who has the brakes system tested, showing the maximum deceleration possible is only 6 ft/s^2. What does this say about how fast the car was traveling at the instant the driver applied the brakes? What should the wheels of justice do to the driver in this situation? Explain.

I throw an arrow into the air, it lands I know where!

We shoot a small steel ball vertically into the air at a speed of 22 m/s in an upward direction, releasing it from ground level.

Build a mathematical model of the altitude of the ball, $s(t)$, in feet at time t in seconds, knowing that it experiences only constant acceleration due to gravity of 9.8 m/s^2.

Keep the model simple and state your assumptions.

a) How long is it until the ball strikes the ground?

b) How fast is it traveling when striking the ground?

c) What is its maximum altitude?

d) How long does it stay at a height equal to 75% of its maximum height? 90% of its maximum height?

e) When is the ball traveling at one half its maximum speed?

f) Over what time interval(s) does the ball have an average velocity equal to 20% of its maximum velocity? Characterize these intervals completely.

Death of an actor[1]

The great detective Sherlock Holmes and his assistant, Dr. Watson, are discussing the murder of actor Cornelius McHam. McHam was shot in the head, and his understudy, Barry Moore, was found standing over the body with the murder weapon in hand.

Let's listen in:

> Watson: Open - and - shut case, Holmes. Moore is the murderer.
>
> Holmes: Not so fast, Watson – you are forgetting Newton's Law of Cooling!
>
> Watson: Huh?
>
> Holmes: Elementary, my dear Watson. Moore was found standing over McHam at 10:06 p.m., at which time the coroner recorded a body temperature of 77.9°F and noted that the room thermostat was set to 72°F. At 11:06 p.m. the coroner took another reading and recorded a body temperature of 75.6°F. Since McHam' s normal temperature was 98.6°F, and since Moore was on stage between 6:00 p.m. and 8:00 p.m., Moore is obviously innocent. Ask any calculus student to figure it out for you.

How did Holmes know that Moore was innocent? Explain.

Elementary (My Dear Watson) Differential Equation[2]

It was hot for this London fall day - 70°F. Holmes arrived at Barker Street Annex to find the inspector hunched over the body. "It is important that we determine the exact time of death, sir, for in that way we may immediately determine the motive," said the inspector. Not wishing to pursue the unpursuable non sequitur, Holmes took out his thermometer and after a few moments of discrete(!) investigation, announced, "I say! 94.6°F. (What, no metric system?) And it is presently noon." With that he departed into the London fog, to return to the body at the same spot in one hour. After performing another investigation Holmes declared, "93.4°F at 1 o'clock." And then silence . . . "Inspector, the murder occurred at exactly 8:58.51204 o'clock a.m. Good day to you, sir!"

Later in their chambers Watson asked, "I say! (The British always say by saying, unlike the Hsitirb's who say by not saying.) How did you do that, Holmes?" "Elementary, my dear Watson.

A simple application of a law of Newton," said Holmes. "Here, here Holmes, you don't mean to say he fell out of a window?" snapped Watson, sensing the gravity of the issue. Remaining cool Holmes began at the beginning, where he always began with Watson, "You see . . . .

REFERENCES

[1] Headlee, C. 2010. Headlee's Math Mansion. http://www.pendragoncove.info/calculus/ch9.htm. Accessed 21 March 2015.

[2] Winkel, B. 1979. Elementary (My Dear Watson) Differential Equations. *Mathematics Magazine*. 52(5): 315.