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STUDENT VERSION
Draining Containers

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Abstract: We examine the question, "Given two rectangular circular cylinders of water with the same volume, but different radii, with a small bore hole of same radius on the center of the bottom through which water exits the cylinder, which empties faster?"

SCENARIO DESCRIPTION

Given two rectangular circular cylinders of water with the same volume, but different radii, with a small bore hole of same radius on the center of the bottom through which water exits the cylinder, which empties faster?

Recall, the differential equation, based on Torricelli's Law[1, p. 3], for the height of a column of water in a constant cross sectional cylinder, $h(t)$, at time t:

$$A(h(t)) \cdot h'(t) = -a\alpha\sqrt{2gh(t)}\,, \tag{1}$$

where $h(t)$ is the height of the column of water at time t; $A(h(t))$ is the cross-sectional area of the cylinder at height $h(t)$; g is the acceleration due to gravity; a is the cross sectional area of the bore hole at the base of the cylinder through which the water exits the cylinder; and α is an empirical number which indicates the percentage of the maximum flow rate which actually gets through the small bore hole. Usually α is on the order of 0.70. Note the negative sign on the right hand side of (1) as water is leaving the cylinder.

REFERENCES

[1] Winkel, B. 2015. 1-015-S-Torricelli.