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STUDENT VERSION
Fish Mixing

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Abstract: This activity gives students a chance to build the underlying differential equation and/or difference equation for a mixing problem using tangible objects (fish) and a student-designed restocking and fishing plan in a lake. The mixture is of two species of fish, one being the current sole species in the lake and the other being introduced via restocking. The students generate data for the mixing problem via a brief game and then conjecture forms of the differential or difference equation model.

SCENARIO DESCRIPTION

A lake in northern Montana is dominated by Arctic Grayling (henceforth called "species A") but the Department of Fish, Wildlife, and Parks is planning to slowly introduce Bull Trout ("species B"). The lake is popular with sport fishermen who remove both species of fish from the lake regularly.

The Department of Fish, Wildlife, and Parks has carefully estimated the number of fish taken by sport fishing each week, and they have decided to keep the fish population as constant as possible, by replacing the fish lost by an equal number of Arctic Grayling and Bull Trout. For example, if there are $N = 50$ fish in the lake at the beginning of the week and fishermen remove $M = 10$ fish during that week, then the fish and wildlife people will restock the lake with 5 Arctic Grayling and 5 Bull Trout. Hence the population of the lake will remain $N = 50$ fish at the end of each week, assuming no new fish are born. Both fish species swim freely throughout the lake and both are targeted by similar bait used by sport fisherman.

In your lake you will use $N =$ _____ and $M =$ _____.

In summary:

- The week starts with $N =$ _____ fish.
- The fish swim freely around the lake.
- $M =$ _____ fish are removed from the lake at random during the week.

- $M =$ _____ fish are restocked at the end of the week. $M/2 =$ _____ of those fish are Arctic Grayling and $M/2 =$ _____ of those fish are Bull Trout.

1. **Conjecture:**
   (a) What do you think will happen to the populations of species A and B over a long period of time?
   (b) Is it possible that species A will be eliminated from the lake with the restocking plan? Explain.

2. **Simulate:**
   (a) Use pennies to represent your N fish and decide with your partner(s) which coin face represents which species (e.g., heads up is species A). Start your lake with 100% species A.
   (b) Decide with your partner(s) how to simulate the swimming of fish, the fishermen, and the Department of Fish, Wildlife, and Parks' restocking plan. Simulate roughly 15 weeks of the fish population representing species A and B with coins. Be sure to let the fish swim thoroughly around the lake and keep track of the proportions of species A and B.

Week #	**Number in population**		**Proportion of population**	
	species A	species B	species A	species B
0			1	0
1				
2				
3				
4				
⋮	⋮	⋮	⋮	⋮

3. **Model:**
   (a) Propose a verbal model for the rate of change of species B in the lake.

   rate at which species B changes = _____ + _____

   (b) Explicitly state any assumptions that you are using in your verbal model.
   (c) Introduce mathematical notation for your proposed model and write your verbal model mathematically. Be sure to include any necessary condition(s).

   model: _____

   condition(s): _____

4. **Analyze:**
   (a) According to your model, what is the long term effect on the fish population in the lake? Use your model to justify your answer algebraically and graphically.

(b) Solve your mathematical model (either numerically or analytically) and compare with your data.

(c) (Extension) Suppose now that the Department of Fish, Wildlife, and Parks does not attempt to keep the population in the lake constant. That is, suppose that fishing reduces the population by M_1 fish each week and the Department of Fish, Wildlife, and Parks restocks M_2 fish each week. Fully explore this scenario.