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Birth of Laplace Transformation Technique

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Abstract: We present a way of introducing the Laplace Transform as the continuous analogue of a power series expression of a function.

STATEMENT

While enjoying an online video lecture (which we highly recommend) "Where the Laplace Transform Comes From," [1, 2] on YouTube, given by Professor Dr. Arthur Paul Mattuck of the Massachusetts Institute of Technology, USA, I was greatly inspired to learn the reason behind the birth of the Laplace Transformation. Professor Mattuck shows how the Laplace Transform definition arises as the continuous analogue of a power series.

This technique is used extensively in order to solve differential equations that arise in many modeling situations of real life. Applications and usefulness of Laplace Transform technique in various scientific and engineering fields through *Mathematica* can be seen in the following reference [3].

A power series about an origin is any series that can be written in the following form:

$$\sum_{n=0}^{\infty} a_n x^n$$

where a_n are numbers and n is a nonnegative integer. One can think of $a_n = a(n)$ as a function of n for each non-negative integer $n = 0, 1, 2, \ldots$. In order to give birth to Laplace transformation technique, we make some associations. The discrete variable n is converted into a real variable t. The coefficient term a_n is written as $f(t)$. The term x^n can equivalently be written as $e^{(\ln x^t)}$. Finally, summation notation can be replaced by its continuous analogue, that is, integration. By doing so, we have following:

$$\sum_{n=0}^{\infty} a_n x^n \rightsquigarrow \int_0^{\infty} f(t) e^{\ln x^t}\, dt\,.$$

For convergence, it is important to have following condition for the above integral:

$$\sum_{n=0}^{\infty} a_n x^n \rightsquigarrow \int_0^{\infty} f(t) e^{\ln x^t} \, dt; \qquad 0 < x < 1$$

$$\sum_{n=0}^{\infty} a_n x^n \rightsquigarrow \int_0^{\infty} f(t) e^{t \ln x} \, dt$$

Since $0 < x < 1$ so it implies that $\ln(0) < \ln(x) < \ln(1)$ or $-\infty < \ln(x) < 0$. Thus $\ln(x)$ has to be negative for the integral to converge, In this regard, we suppose $\ln(x) = -s$ where $s > 0$. Thus, the final integral takes the form:

$$\sum_{n=0}^{\infty} a_n x^n \rightsquigarrow \int_0^{\infty} f(t) e^{-st} \, dt = F(s); \quad s > 0$$

.

In this way, we can say that Laplace Transform is simply stretching a discrete (infinite series) into a continuous (integration) analogue.

REFERENCES

[1] Mattuck, Arthur. 2008. Where the Laplace Transform Comes From (Part I). https://www.youtube.com/watch?v=zvbdoSeGAgI.

[2] Mattuck, Arthur. 2008. Where the Laplace Transform Comes From (Part II). https://www.youtube.com/watch?v=hqOboV2jgVo.

[3] Winkel, Brian. 2015. 7-005-T-Text-LaplaceTransformOverView.