STUDENT VERSION
MODELING A NONLETHAL INFLUENZA EPIDEMIC

STATEMENT

A boarding school is a relatively closed community in which all students live on campus, teachers tend to live on or near campus, and students do not regularly interact with people not in the boarding school community. Table 1 gives data for an influenza outbreak at a boarding school in England during which there were no fatalities. These data were compiled from the Communicable Disease Surveillance Center [1, p. 587] and are given as an example by Murray in his book on mathematical biology [2]. The data values were extracted from the graph found in [3].

Time (in days)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of Bedridden Boys	1	3	25	72	222	282	256	233	189	123	70	25	11	4

Table 1. Total Number of Bedridden Boys on Day t.

There were 763 boys at the English boarding school from which our data was obtained. We can see from Table 1 that the initial number of bedridden students is one. The data given is number of bedridden boys rather than number of boys with influenza; let us assume that the number of boys who are bedridden on day t consists exactly of all those who are newly infected and all continuing infections (students who are still on bedrest after being infected on a prior day), namely that no boys are bedridden for any other reason during this period of time and also that all boys with influenza will be not only symptomatic but bedridden[1]. These assumptions, together with excluding teachers and staff from the population, imply that the number of students infected on day zero is one, or, $I(0) = 1$, where $I(t)$ is the number of students symptomatic on day t (either infectious or infected).

1. Plot the data points of the number infected students on day t against time in days, t. Comment on what you see and relate it to how you expect influenza to spread in a small, semi-closed, community of healthy, school-age children. Recall that influenza is spread primarily through

[1]One may wish to test the effects of this last assumption on the model in the analysis phase. Another perhaps more realistic assumption is that the bedridden boys represent a fixed percentage of the infected students.

social contact, but moisture droplets containing the virus can also contaminate shared surfaces, so influenza can be transmitted from one person to another without direct contact.

2. To track an epidemic we need to know how many people are susceptible, how many are infected, and how many have recovered (and are therefore, in the case of a virus such as influenza, immune). Build a differential or difference equation model of $S(t)$, the number of susceptible students on day t, $I(t)$, the number of infectious or infected students on day t, and $R(t)$, the number of recovered students on day t, for $t = 0, 1, \dots, 13$. Defend each term in each equations, and define the parameters and their units.
3. Play with the parameters in the model you created in (2) to see which parameter values might produce the behavior demonstrated in the plot of the given data you constructed in (1). Are all of those parameter values reasonable?
4. Adjust the parameters to find a best fit to model the data. Remember to define your criterion for "best fit."
5. Confirm or validate that your model is reasonable. Conduct a sensitivity analysis for at least one of your assumptions.
6. Compare the values of your model from (4) with the data. Use all available and appropriate techniques when assessing your model, including quantities that can be computed, such as sum of squared error and coefficient of determination and also other comparisons, such as the predictive or visual power of the model. How well does the general shape of the model fit the data? Assess and comment upon the fit of your model and on discrepancies between your model and the data. Consider whether your model predicts infections late or early (compared to the data) and what consequences this might have for an attempt to manage an epidemic. Conduct a sensitivity analysis for one of your parameters.

REFERENCES

[1] Communicable Disease Surveillance Center. 1978. News and Notes: Influenza in a Boarding School. *British Medical Journal*. 1(6112): 586-590. Accessed 25 October 2014.

[2] Murray, J. 1993. *Mathematical Biology, 2nd, Corrected Edition*. New York: Springer-Verlag.

[3] Switkes, J. 2003. A modified discrete SIR model. *The College Mathematics Journal*. 34(5): 399-402.