Instructor Notes

This worksheet is meant for use in a typical Calculus III (Multivariate Calculus) course during the coverage of arc length and curvature (e.g. Stewart's 9th ed.[1] Section 13.3, OpenStax Calculus 3[2] Section 3.3, or Active Calculus – Multivariable[3] Section 9.8). Its two main goals are:

(a) Show the students a real-world application of the otherwise abstract concept of *curvature*.

(b) Give the students some experience with numerical computations, and working with functions defined by discrete data rather than closed-form formulas.

It is designed to be accessible with very little programming background, and can be completed either using a programming language or a spreadsheet program (links to free options[4] [5] are included in the worksheet document). The final deliverable and submission format are not specified and are left up to the instructor, but it is expected that the students will submit a brief report explaining what they did, their final result, along with any code they used.

The track data and formula for $v_{\max}$ are both artificial, although it should be noted that many of the formulas used by real transportation engineers in setting maximum permissible speeds are similar in format to the one provided in this worksheet. While the worksheet does not mention it, in class you may also wish to discuss the concept of *radius of curvature*, which is the reciprocal of curvature.

The following notational conventions are used in the worksheet:

- A lowercase kappa, κ, denotes curvature
- Vector arrows, $\vec{r}$, are used to distinguish vector variables and functions
- Angle brackets, $\langle a, b, c \rangle$, are used to enclose vector elements
- Double bars, $\|\vec{u}\|$, denote vector magnitude

If your class uses different notational conventions, it is recommended that you edit the worksheet's source TEX file for consistency.

What to Give to the Students The students should be provided with the worksheet file `Numerical Curvature Handout.pdf` along with the track data file `track.tsv`. You may also wish to provide a brief tutorial for the programming language or spreadsheet program to be used by the students.

Solutions Using central differences to approximate all derivatives, the maximum curvature is approximately $\kappa \approx 0.14542$ m^{-1}, for a maximum permissible speed of $v_{\max} \approx 7.01643$ m/s. Numerical answers may differ slightly if different numerical approximations (e.g. forward differences or backward differences) are used, but not by much.

[1] `https://www.cengage.com/c/calculus-9e-stewart/9781337624183PF/`
[2] `https://openstax.org/details/books/calculus-volume-3`
[3] `https://activecalculus.org/ACM.html`
[4] `https://www.google.com/sheets/`
[5] `https://www.libreoffice.org/discover/calc/`