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**Abstract:** Differential equation models are frequently used to describe non-linear trajectories of longitudinal data. This study proposes a new approach to estimate the parameters in differential equation models. Instead of estimating derivatives from the observed data first and then fitting a differential equation to the derivatives, our new approach directly fits the analytic solution of a differential equation to the observed data, and therefore simplifies the procedure and avoids bias from derivative estimations. A simulation study indicates that the analytic solutions of differential equations (ASDE) approach obtains unbiased estimates of parameters and their standard errors.

Compared with other approaches that estimate derivatives first, ASDE has smaller standard error, larger statistical power and accurate Type I error. Although ASDE obtains biased estimation when the system has sudden phase change, the bias is not serious and a solution is also provided to solve the phase problem. The ASDE method is illustrated and applied to a two-week study on consumers' shopping behaviour after a sale promotion, and to a set of public data tracking participants' grammatical facial expression in sign language. R codes for ASDE, recommendations for sample size and starting values are provided. Limitations and several possible expansions of ASDE are also discussed.

**Keywords:** parameter estimation, simulation, fitting, shopping, behaviour