Sontag, Eduardo D. 2018. Lecture Notes on Mathematical Systems Biology. 305 pp.

See http://www.sontaglab.org/publications.html. Accessed 20 March 2020.

The first version of these notes dates back to 2005. They were originally heavily inspired by Leah Keshet's beautiful book *Mathematical Models in Biology* (McGraw-Hill, 1988), and much material was "borrowed" from there. With time, the emphasis turned to be heavier on "systems biology" ideas, and lighter on traditional population dynamics and ecology. The goal was to provide students with an overview of the field. With more time, one could include far more material, such as Turing pattern-formation, detailed tissue modeling, cancer progression and resistance, and synthetic biology.

Starting with Version 6, there is also a first very short chapter on difference equations, which can be skipped without loss of continuity. (And, conversely, can be covered by itself.) This chapter allows students to understand notions of dynamics, including fixed points, stability, periodic orbits, and chaos, in a very simple graphical context. The material in this chapter is largely "borrowed" from part of Chapter 1 of Elizabeth S. Allman, John A. Rhodes's *Mathematical Models in Biology: An Introduction*, (Cambridge University Press, 2004). The reader should consult that text for much more material, including vector systems of difference equations and for applications to molecular evolution, phylogenetic trees, classical genetics, and epidemics.

The writing is in places "telegraphic" and streamlined, so as to make for easy reading and review. (The style is, however, not consistent, as the notes have been written over a long period.) Furthermore, the notes do not employ "definition/theorem" rigorous mathematical style, so as to be more "userfriendly" to non-mathematicians. However, the reader can rest assured that informally stated facts can be converted into theorems. Also, I tried to focus on intuitive and basic ideas, as opposed to going deeper into the beautiful theory that exists on ordinary and partial differential equation models in biology – for which many references exist.

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Originally, the deterministic chapters (ODE and PDE) of these notes were prepared for the Rutgers course Math 336, Dynamical Models in Biology, which is a junior-level course designed for Biomathematics undergraduate majors, and attended as well by math, computer science, genetics, biomedical engineering, and other students. Math 336 does not cover discrete methods (genetics, DNA sequencing, protein alignment, etc.), which are the subject of a companion course.

With time, the notes were extended to include the chapter on stochastic kinetics, covered in Rutgers Math 613, Mathematical Foundations of Systems Biology, a graduate course that also has an interdisciplinary audience. In its current version, the material no longer fits in a 1-semester course. Without the stochastic kinetics chapter, it should fit in one semester, though in practice, given time devoted to exam reviews, working out of homework problems, quizzes, etc., this is unrealistic.

Pre-requisites for the deterministic part of notes are a solid foundation in calculus, ideally up to and including sophomore ordinary differential equations, plus an introductory linear algebra course.

Students should be familiar with basic qualitative ideas (phase line, phase plane) as well as simple methods such as separation of variables for scalar ODE's. However, it may be possible to use these notes without the ODE and linear algebra prerequisites, provided that the student does some additional reading. (An appendix provides a quick introduction to ODE's.) The stochastic part requires good familiarity with basic probability theory.

I am often asked if it is OK to use these notes in courses at other universities. The answer is "of course!" though I strongly suggest that a link to the my website be provided, so that students can always access the current version. And please provide feedback!