Sethi, Suresh. 1972. Optimal Control of the Vidale-Wolfe Advertising Model. *Operations Research*. 21(4). Preprint.

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Abstract: This paper considers an optimal-control problem for the dynamics of the Vidale-Wolfe advertising model, the optimal control being the rate of advertising expenditure to achieve a terminal market share within specified limits in a way that maximizes the present value of net profit streams over a finite horizon. First, the special polar cases of fixed and free end points are solved with and without an upper limit on advertising rate. The complete solution to the general problem is then constructed from these polar cases. The fixed-end-point case with no upper limit on the advertising rate is solved by using Green's theorem, while the other cases require additional use of switching-point analysis based on the maximum principle. The optimal control is characterized by a combination of bang-bang, impulse, and singular control, with the singular are forming a turnpike.

Introduction.

As one of the earliest management science applications to marketing, Vidale and Wolfe developed a simple model of the sales response to advertising that was consistent with their experimental observations. In the fifteen years since its inception, the model has had a wide appeal because of its extreme simplicity and capacity to capture the interaction between advertising and sales in an intuitively satisfying manner.