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Abstract: This paper focuses on predator-prey models with juvenile/mature class structure for each of the predator and prey populations in turn, further classified by whether juvenile or mature individuals are active with respect to the predation process. These models include quite general prey recruitment at every stage of analysis, with mass action predation, linear predator mortality as well as delays in the dynamics due to maturation. As a base for comparison we briefly establish that the similar model without delays cannot support sustained oscillation, but it does have predator extinction or global approach to predator-prey coexistence depending on whether the ratio α of per predator predation at prey carrying capacity to the predator death rate is less than or greater than one.

Our first model shows the effect of introducing an invulnerable juvenile prey class, appropriate, e.g., for some host-parasite interactions. In contrast our second model shows the effect of limiting predation to a prey juvenile class. Finally, in a third model we consider an inactive juvenile predator class, which would be appropriate for many traditional situations in which the generation time for the predator is significantly larger than that of the prey. In all cases the introduction of a juvenile class results in a system of three delay-differential equations from which the two equations for the mature class and the nonstructured class can be decoupled. We obtain some global stability results and identify a parameter α, similar to the α of the unlagged model, which determines whether or not the predator is driven to extinction. With $\alpha > 1$, and considering the maturation age of the juvenile class as a bifurcation parameter, we obtain Hopf bifurcations in our second and third models, while in the case of juvenile prey (in the first model) the unique coexistence equilibrium remains stable for all positive delays. Although the delay is "physically present" in all three models, we obtain scaled, nondimensional replacement models with that physical presence scaled out. After analyzing the scaled equations we show that all our results hold for the original models. We pursue the bifurcation in the inactive juvenile predator model with numerical simulations. Strikingly similar results over a variety of birth functions are observed. Increases of the maturation delay first produce Hopf bifurcation from steady state to periodic behavior. Even further increase in the delay produces instabilities of the bifurcating periodic solutions with corresponding interesting geometry in a two-dimensional plot of period vs. delay.

Keywords: predator-prey, host-parasite, age structure, maturation delay, delay differential equations, bifurcation