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From the opening,

“Recently, there has been sustained interest in the use of partial differential equation (PDE) models to obtain insight into biological and sociological phenomena. This work is separated into two parts were we study the well-posedness two such PDE systems.

“In particular in the first part we study a fully-parabolic system of PDEs for residential burglary ‘hotspots’ (spatio-temporal areas of high density of crime). Although crime is a ubiquitous feature of all societies, certain geographical locations have a higher propensity to crime than others. In fact, residential burglary data exhibit areas of high crime density surrounded by areas of low crime density. There have been many studies indicating that the “repeat and near-repeat victimization effect,” which states that crime in an area induces more crime in that and neighboring areas, leads to the residential burglary hotspots seen in real data. Short et al. develop in [92] an agent-based statistical crime model whose dynamics rely on this repeated victimization effect. The formal continuum limit of this model is a nonlinear couple system of PDEs. This models exhibits the right qualitative behavior (with certain parameters), that is, the existence of crime ‘hotspots’. In this work we are concerned with the existence and uniqueness of solutions of this model. In two-dimensions we prove local existence and uniqueness of classical solutions with Hm-initial data on periodic domains. Furthermore, we prove a continuation argument that provides a sufficient condition for global existence of the solution. More specifically, we determine a controlling quantity; hence, the solution continues to exist while this quantity remains bounded. Finally, motivated by the relation between this PDE system and the Keller-Segel model for chemotaxis (the movement of cellular organisms in response to some chemical concentration in their environment), we conclude this section by studying a modified model; which provides insight into the global theory of the original model.

“In the second part we study an aggregation equation with degenerate diffusion. Aggregation equations have been studied for a wide variety of biological applications in migration patterns in ecological systems and Patlak-Keller-Segel models for chemotaxis. We study the local and global well-posedness of weak-solutions of an equation that models the competition between aggregation (modeled via a convolution with a kernel) and over-crowding effects (modeled via general degenerate diffusion). We divide they system into three types: subcritical, supercritical, and critical. We prove global existence for subcritical problems, which correspond to the diffusion dominating aggregation. We prove finite time blow-up for a subclass of supercritical problems, which correspond to the aggregation dominating the diffusion. Finally, we show that there is a critical mass phenomena for the critical problems, which correspond to the aggregation and diffusion balancing out."

Keywords: partial differential equations, model, crime, biology, diffusion