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https://pages.pomona.edu/~ajr04747/Spring2017/Math102/Notes/Math102Spring2017Notes.pdf
. Accessed 29 March 2023.

From the Preface

Differential equations are ubiquitous in the sciences. The study of any phenomenon in the physical or biological sciences, in which continuity and differentiability assumptions about the quantities in question can be made, invariably leads to a differential equation, or a system of differential equations. The process of going from a physical or biological system to differential equations is an example of mathematical modeling. In this course, we will study differential equations in the context of the modeling situation in which they appear.

The construction of a mathematical models involves introducing variables (usually functions) and parameters, making assumptions about the variables and relations between them and the parameters, and applying scientific principles relevant to the phenomenon under study. The application of scientific principles usually requires knowledge provided by the _field in which the study is conducted (e.g., physical laws or biological principles). In a lot of cases, however, there is set of simple principles that can be applied in many situations. An example of those principles are conservation principles (e.g., conservation of mass, conservation of energy, conservation of momentum, etc.). For all the situations we will study in this course, application of conservation principles will suffice.

We will begin these notes by introducing an example of the application of conservation principles by studying the problem of bacterial growth in a chemostat. We will see that the modeling in this example leads to a system of two differential equations. The system obtained will serve as an archetype for the kind of problems that come up in the study of differential equations.

The chemostat system will not only provide examples of differential equations; but its analysis will also serve to motivate the kinds of questions that arise in the theory of differential equations (e.g., existence of solutions, uniqueness of solutions, stability of solutions, and long-term behavior of the system).

Keywords: linear systems, Lotka-Volterra, pendulum, model, differential equations, existence, uniqueness