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CourseSource

Lesson

Learning How to Make "Good Enough" Estimations

Jon D. H. Gaffney[1*]

[1]Physics Department, Utica University

Abstract

The ability to estimate physical quantities is a useful skill that can help develop critical thinking and scientific literacy. This lesson provides an accessible way to teach students how to estimate physical quantities by focusing on three key aspects: making assumptions based on previous experiences, explicitly converting units, and communicating the solution clearly. Students are first taught how to perform each of these three steps through a problem-based interactive learning cycle led by the instructor before performing further estimates in groups during a minimally structured investigation. Because this lesson is lean on physics content, it can be used in a variety of classes and is ideal as an early-term laboratory activity. Students engage with the activity by investigating spaces, planning an approach to a solution, and ultimately presenting that solution to one another.

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*__Correspondence to:__ jdgaffney@utica.edu

Learning Goals

Students will:

◊ begin to understand how to perform estimations.

◊ understand how to convert units.

◊ understand why estimating physical quantities can be useful.

Learning Objectives

Students will be able to:

1. use measurement techniques to estimate physical quantities when exact measurements are not feasible.
2. incorporate their previous experiences into estimations of physical quantities (such as length, area, volume, mass, weight, speed, and flow rate).
3. convert units explicitly when provided with conversion factors.
4. communicate their problem solutions clearly.

INTRODUCTION

Being able to estimate physical quantities is useful in many ways. In addition to its use as a tool for evaluating whether claims are reasonable, estimating also helps students build upon their prior experiences to quantify the world and develop some intuition about scale (1). While these abilities often support success in science and aspects of everyday life, explicit estimation lessons are not commonplace. Fermi, who was known for making back-of-envelope calculations to answer seemingly impossible questions, seemed to unskilled estimators to have performed magic. He achieved his results by breaking problems down into smaller, manageable pieces and combining the simpler approximations into a single estimate (2). These skills can be taught. Estimation problems can be used to demystify scientific processes such as scaling, bounding the problem with extreme cases, and performing dimensional analysis (3). Learning how to estimate can support scientific literacy by driving home the point that the physical world is measurable and by supporting complex problem solving across disciplines (4). Furthermore, being able to estimate can be empowering. Students who can perform estimates can gain more confidence in both their ideas and their ability to test those ideas.

Due to the importance of developing these scientific skills, some instructors choose to weave estimation explicitly through their courses by regularly including estimation problems in homework, class exercises, and exams; doing so may be especially helpful in physics courses taken by budding scientists such as algebra-based physics for life sciences (1). Significant resources exist for sample Fermi problems as well as ways to implement those throughout the course (for example, Using math in physics - Estimation and Teaching Physics with the Physics Suite). A strength of such an approach is that as students learn new science content, the set of estimation problems they can access and solve also grows, and practicing estimation techniques within new content areas subsequently reinforces understanding of that content.

However, weaving estimation into an entire course can be a considerable commitment. Additionally, many students, especially those not majoring in a STEM discipline, have little content knowledge in physics or other sciences to draw from at the start of a semester. Therefore, even if instructors are only occasionally interested in asking students to perform estimates, they need to explicitly teach students how to do so (5). A plethora of good examples of estimation problems that are largely content-free but rich in context can be found in the ongoing column *Fermi Questions* in *The Physics Teacher*. While these are excellent problems, there is no guidance provided for instructors looking to use them instructionally.

In addition to the resources listed above, there are others available for teaching estimation techniques within specific contexts. One specific example is a lesson for comparing various techniques for estimating the population of squirrels (6). Another lesson utilizes the context of film scenes for generating estimation problems (7). Classic estimation questions can also be enhanced by requiring students to provide a range within which they expect to find the quantity, thus creating a friendly competition (8). This gamified approach is useful for teaching about the separate important roles of accuracy and precision in scientific work. However, these lessons may be too specialized for an instructor seeking to teach an introductory lesson about performing estimates that are quick, reasonable, and relevant to interactions with everyday physical objects.

The modular lesson plan presented here can be used largely independently of physics content knowledge while providing an introduction to estimation as a skill set. Students make estimations about measurable physical quantities, receiving instructor feedback that guides them to become more explicit about listing their assumptions, converting units, and connecting their answers to their everyday experience. Since the lesson is an introduction to estimation, it can stand alone or be used as a starting point for further content-rich estimation problems such as those discussed above.

Intended Audience

This lesson was developed for use early in an introductory algebra-based physics course, *Technical Physics*, which is the first in a two-semester analytical physics course sequence designed especially for Construction Management (CM) students. Students enrolled in *Technical Physics* often have little background in science and mathematics, but have considerable experience with construction techniques and are often quite skilled with kinesthetic tasks.

In *Technical Physics*, the lecture and lab are integrated as the class sessions are 110 minutes each, three times per week, in a large room with moveable tables and easily-accessible lab equipment, reminiscent of SCALE-UP (Student-Centered Active Learning Environment for Upside-down Pedagogies) classrooms (9, 10). There are approximately thirty students enrolled per section, and the course is taught by both a lead and a secondary instructor. This lesson takes advantage of the integrated classroom style by alternating between student activity and instructor-led discussion before culminating in a laboratory experience. This lesson is therefore best suited for the flexibility of that environment.

Nonetheless, this lesson can easily be adapted to a course taught in a lecture hall, with or without a separate laboratory section. The first part of the lesson focuses on one to three estimation problems. The second portion can be conducted during a lecture or a separate lab section. Alternatively, because the students are actively engaged in measurement and estimations, the activity could be adapted to run synchronously online. I do not recommend using this lesson asynchronously online, as real-time conversations are key components to the instruction. Because of the low content expectations and its use as an early-semester activity, it is appropriate for use in any introductory-level physics course regardless of the enrolled students' fields of study.

Required Learning Time

This lesson was designed to take approximately four hours to implement, not including post-lab assessments. Additional time could be allotted for including more instructional cycles if necessary. The lab can also be expanded to include more estimation questions. See Table 1.

Prerequisite Student Knowledge

This lesson is particularly useful early in the semester, because it does not rely on much specific physics content. However, students are expected to know how to find the area and volume of objects; be able to calculate rates; understand scientific notation, exponents, and fractions; and understand the role of units in describing quantities. A central emphasis of the lesson is learning how to convert units; dimensional analysis relies on being able to manipulate mathematical expressions to achieve a target quantity. The process of doing so is explicitly taught in this lesson, but students must understand what is meant by a "physical quantity" and the role that units play. Because we could not assume that entering students had this knowledge, we divided our students into eight groups of three or four students prior to this lesson, and each group learned about and then presented to the rest of the class one of eight important physical quantities: length, area, volume, mass, force, energy, pressure, and power. As an alternative to "power," students could have explored the physical quantity "time" or "time rates" in general.

Students should also have proficiency in performing measurements. The estimation problems provided in this lesson only require the use of meter sticks and triple-beam balances, but the activity could be extended to include other tools such as calipers or micrometers.

Prerequisite Teacher Knowledge

I recommend that instructors familiarize themselves with estimation techniques and practice solving a variety of Fermi Questions to become comfortable with the process used (11). The concept of benchmark or landmark numbers is also helpful (for example, this video by Stand-Up Maths). Instructors should also understand **why** it is necessary and important to be explicit when teaching how to solve estimation problems (5), in addition to understanding how students can directly benefit in their fields of study by learning estimation techniques. To gain this insight, instructors can either ask experts in those fields or consider asking the students themselves (for example, see the post-activity assessment).

SCIENTIFIC TEACHING THEMES

Active Learning

This lesson was designed for a classroom that is highly interactive and technology-rich, based on SCALE-UP models of similar classrooms (9, 10). As such, it utilizes group work extensively. The lesson itself follows a problem-based learning approach that centers students' attempts to solve problems to learn the material (12). Here, students learn how to perform estimations through a scaffolded set of practice problems. Students work in groups of three or four throughout the lesson, discussing and presenting their ideas (13). They decide how to solve the problems through discussion with the group, they present solutions to the class, and they propose critiques of solution ideas in classroom discussions. During the lesson, students use group roles to facilitate engagement with the material.

Assessment

During the Interactive Lecture Cycle, the students are constantly providing formative feedback to the instructor. That feedback allows the instructor to steer the classroom discussion appropriately and permits the instructor to increase or reduce the number of examples based on how the class's overall learning seems to be progressing. For example, if the students are having difficulties performing explicit unit conversions, instructors can introduce direct practice problems (for example, converting kilometers per second to miles per hour) for them to hone those skills.

During the laboratory portion of this activity, students are expected to reflect on their own understanding and to ask questions as they work. The instructor also frequently collects feedback by asking the groups questions about how they approached and answered the questions. This feedback influences the subsequent group discussions at the conclusion of the laboratory portion of the activity.

Students complete a pre-post assessment to determine how this activity improved their estimation skills. They estimate the number of gallons of air in the room both before and after the lesson, without explicitly discussing the problem during the lesson. Students also answer an estimation question for credit in the course, and their written answers to the laboratory questions are graded for course credit as well. Evaluation of these assessments is based upon the process more than the answer, with the focus being on students clearly articulating their assumptions and approach, explicitly performing any unit conversions, and presenting a sensible answer.

Inclusive Teaching

This lesson is accessible to the students in a variety of ways. Students are able to directly encounter many of the physical objects they are asked to approximate. Other objects are represented in different ways, such as by overhead map. Students themselves are asked to depict their estimations visually with sketches, mathematically with calculations involving unit conversions, and verbally through written and oral communications of their process and solution. Because certain distances can be "paced out" in various ways, students with visual impairments can utilize other tools to achieve the same results. Similarly, students with ambulatory challenges are able to use the pictorial representations such as maps that are provided to all students. The instructor can easily provide materials such as photographs taken of buildings or other spaces as well.

Another accessibility strength of this lesson is in how its timing is managed. Although certain phases of the interactive lecture cycle are precisely controlled, others—such as how students approach and solve the individual tasks including the laboratory exercises—are not. The flexibility of the timing allows students to work at their own pace, cycling through the problems if they choose not to work through them in order. They are able to take breaks and ask questions of the instructor as needed, catering both to students with intense focus and those who prefer to bounce between tasks.

Finally, students work in groups to ensure that each student can contribute according to their strengths and learn from other students. Because the starting point for all estimations is to relate the quantities in terms of personal experiences, students are invited to bring their own history into the problems. For example, one student may compare the size of campus to the farm she grew up on; another may relate the flow rate of a faucet to the showerhead he recently bought for a bathroom renovation. In the small groups, students are provided with the opportunity to bring their identities and experiences into the problems and use those as the basis for their sensemaking.

LESSON PLAN

Students begin this lesson by completing an estimation pretest. Individually, students determine how many gallons of air are in the room (Supporting File S1). These estimates are collected without discussion and are compared with student responses after the activity is completed. The lesson itself consists of two halves, each requiring about 90–120 minutes: the interactive learning cycle and the laboratory activity. These were designed to be completed in a studio environment, but they should also be effective in lecture halls and laboratory settings.

Interactive Learning Cycle

The interactive learning cycle provides problem-based direct instruction for the process of making rapid estimates of physical quantities. The emphasis of this instruction is to generate quick, reasonable estimates by explicitly recording assumptions, converting units carefully, and properly reporting the answer.

Assigning Groups

Prior to instruction, assign students to small groups of three or four students each. These can be the same as existing long-term groups in the course, or they can be created ad hoc for this activity. It is recommended to briefly introduce students to the idea of group roles and assign these to the students, informing them that they would rotate through the roles during the activity and from one lab to the next throughout the semester (Supporting File S2). We added the role of presenter to the group roles of manager, recorder, and skeptic as suggested by Heller and Hollabaugh (13). Specifically, for this task, the skeptic is empowered as the only group member to ask questions of the instructors, and the presenter is expected to be prepared to represent the group's work to the class if asked. In cases where there are only three group members, the presenter role is dropped and the remaining team members cover those duties. Group roles are rotated after each problem during the learning cycle so that each group member gets to experience multiple roles.

Interactive Problem-Based Approach

The interactive learning cycle consists of three estimation questions, followed by lecture and discussion. More questions could be added if necessary. Students have 10 minutes to discuss each question within the group. Each group is provided with a meter stick. They are permitted to walk around the room and measure anything they desire. They should write their answer and supporting work on a whiteboard provided to them (roughly 16-inch by 16-inch square cuts of white tile board, obtainable at large hardware stores) and post their result onto a Google Sheet (Supporting File S3) that can be shared and posted via projector for students to see. While the students are working on the problems, the instructor observes student discussions, making notes about common themes for bringing up in discussion later. Additionally, the instructor can discern some interesting solutions to share with the class. Call on a randomly selected group (I find that rolling a die works well). The presenter of the chosen group shares their whiteboard with the class. Other groups are then given the opportunity to present theirs as well. Discuss the lowest and highest estimates that groups reached (as seen in the Google Sheet), highlighting assumptions and identifying any possible errors in calculation. Building off of these efforts, the instructor models what a finished solution should look like and allows further discussion. At the conclusion of that discussion, repeat the cycle with a new question.

Highlighting Learning Objectives

The three questions in this interactive learning cycle emphasize three key learning objectives that represent important aspects of estimating: incorporating assumptions, explicitly converting units, and clearly presenting and communicating an estimate for the question.

In the first question, students determine how long it would take to walk to a particular destination thousands of miles away (Supporting File S1). The instructor-led discussion can focus on the assumptions that students made. Some frequent assumptions include particular walking speeds, whether the straight-line distance is a reasonable estimate, the difficulty in crossing certain terrain (like mountains or rivers), how many hours per day someone can walk, how much weight they need to carry on their backs, etc. Students may ask what they **should** assume. Tell them to draw upon their own experiences and to build from what they know, in consultation with their group. In the class-wide discussion, centralize those decisions. Typically, groups provide responses that vary substantially, by a factor of ten or more. Practice consensus-making to decide as a class whether certain assumptions are reasonable. Additionally, remind the students that variation is expected.

In the second question, students estimate how many tennis balls fit into a mid-sized sedan. They are handed a tennis ball, but not a mid-sized sedan, for reference. This question will naturally lead to discussions about unit conversions, especially discussions about how to convert volumes, such from cubic centimeters (the measure of a tennis ball using a meter stick) to cubic feet (a natural unit for many students when estimating the length times width times height of a rectangular prism). Additionally, a common question is how to estimate the volume of a sphere. A quick comparison between the "true" volume and the volume of a cube with an edge equal to the diameter of the tennis ball reveals that the cubic volume is a "good enough" approximation, especially when considering how to pack spheres into a space. Finally, this question typically generates a wider range of responses than the first, so it is a good opportunity to emphasize the importance of being explicit about unit conversions, rather than just doing them in a calculator or scribbling notes. Those explicit calculations can more easily be checked by the members of the group, especially when estimates feel "off."

The third question asks students to determine how many sheets of paper would cover the hallway outside the classroom. They are only given one sheet of paper along with their meter stick. This question typically yields less variation, so the instructor can focus the conversation on how to clearly communicate the entire process, not merely the result. For example, in our case the students needed to decide where the hallway started and stopped, as well as its size as it widens and narrows in places. Additionally, students may use different methods of estimating the length of the hallway (for example, by pacing or counting the ceiling tiles). In the discussion, remind the students that their choices need to be written down as part of the solution.

Laboratory Activity

Provide the students with a link to the three-problem lab activity on Google Drive (Supporting File S4), and allow them to work at their own pace after determining their group roles. Students do not need to work the estimation problems in any particular order. They should update the provided Google Sheet with their estimates (Supporting File S5). Making the Sheet available to the students allows them to see how other groups are progressing and can provide subtle feedback about their estimates (for example, alerting a group that their estimate is significantly low). The assigned problems should encourage the students to leave the classroom to take measurements, so permit students to leave as long as they return at the conclusion of the allotted time period (approximately 60–75 minutes). Remain in the room to address any questions.

The laboratory activity is comprised of the following three questions, which are meant to be adapted for each campus or other local setting so that students can answer questions about familiar locations: 1. How many blades of grass are on the Burrstone Campus of Utica University? Students are provided a campus parking map for reference. 2. How many bricks are in the Construction Management building? 3. If Thurston Hall were sealed up and one sink on the second floor left to run, how long would it take for the building to entirely fill up with water?

While the students plan and complete their activity, display the spreadsheet in the classroom and answer student questions. Typically, students ask about what technology or measuring equipment they are permitted to access. Students may only access the internet for unit conversions. They should not access blueprints or use tools that provide a precise measurement. After about a half hour, take a more active role in circulating among students and probing with questions such as, "What is your plan to determine the size of campus?" or, "How did you determine the number of blades of grass in a cubic inch?" Allow prior experience (*e.g.,* "it's about twice the size of the farm I grew up on") but not overly vague reasoning. Provide suggestions for improvement, such as, "instead of guessing, how might you measure that?" or, "that's a creative approach; what unit conversions will you need?" Critically, continue to remind students to record their assumptions and process, to explicitly show their work converting units, and to prepare effective communication to answer each question.

Visit each group to inquire about their approaches, focusing questions on estimates that seemed especially large or small. After these small group discussions, call on one or two groups to present their results. Call on different groups for each of the three estimates. Alternatively, each group can be tasked with posting solutions on whiteboards for a "gallery walk" (14). In either case, the purpose of the presentation is to allow the students an opportunity to present and consider modifications to their estimations. During this presentation time, groups can also discuss their level of uncertainty and use each other's answers to discuss how to report their estimates so that their uncertainty is properly reflected, for example, through the use of significant figures. Such a discussion could be powerful if one of the instructor's learning objectives when using this activity is for students to learn about uncertainty or significant figures. Make sure to allow sufficient class time for the groups to debrief.

Provide students with a post-lab assignment that includes several metacognitive questions about their experience working in their group as well as three follow-up questions about the estimates they made (Supporting File S6). Students should complete this activity individually to help them process and practice the skills developed in class. Additional estimation questions that build off of the work done in class such as, "How long would it take to mow all of the grass on campus?" could be added and would provide further practice. Questions can also probe whether students have gained insight or understanding about the relevance or value of estimates. For example, "provide an instance where being able to generate a quick, good enough estimate is a useful skill in your major," can give students an opportunity to think carefully about why this activity is important to them.

Post-Activity Assessment

Assess student learning by providing a post-test that is the same question that they used at the beginning of the unit: How many gallons of air are in this room? After collecting student responses, provide a response that would receive full credit in subsequent assessments (Supporting File S7), emphasizing once again the importance of showing all work.

TEACHING DISCUSSION

Lesson Effectiveness

There was a high level of student engagement with the activity. Students made good use of their time to consider their estimates, especially during the laboratory portion. In one case, as the students were wandering a classroom building, trying to figure out how long it would take to fill with water, they encountered one of their construction management instructors. The students began asking him questions, leading to a robust conversation about material stress, pressure, and building design. The professor remarked that he was impressed that the students were so interested in the lab questions that they wanted to know more about their building, and student responses to that question sometimes explored the idea that, for example, the windows would likely blow out before the building filled.

The human-subject research was approved by Utica University's Internal Review Board and received exempt determination under the category of Educational Research. By looking at the pre/post question asked in the activity, we can determine that students improved with respect to the learning objectives. However, it was clear that this activity served as an introduction and that further practice would be needed to produce mastery. More students showed work to communicate their response when answering the question (Learning Objective #4, 81% up from 55%). Similarly, more students provided some clear attempt to convert units explicitly (Learning Objective #3, 67% up from 24%). Because students generally made sensible estimations of the dimensions of the room in familiar units (feet), explicitly showing their work and attempting the unit conversions to gallons helped expose particular errors. For example, students sometimes divided rather than multiplied the number of gallons per cubic foot, or performed one- rather than three-dimensional unit conversion (for example, 12 inches per foot rather than 1728 cubic inches per cubic foot). Those errors permit on-the-fly instructional interventions, while a list of numbers or an answer without context would not provide enough information for this. Indeed, there were zero pure guesses (where a student simply wrote down a number) on the post-test, compared to five on the pretest (approximately 17%).

Progress on learning objectives 1 and 2 was gauged mostly by observation as students worked through the lab activities following the interactive learning cycle. When asked about their methods, students frequently relied upon previous experience to provide approximate dimensions (for example, comparing the size of campus to an athletic field). However, for questions such as the one that involved flow rate, some students needed to be explicitly reminded that they could perform a quick measurement or come up with a number based on their experience with replacing faucets or showerheads. This activity was effective at allowing those moments of discussion to happen between instructor and students, so that students could develop their skillset as they worked to improve their estimates.

Possible Adaptations

This lesson was originally designed for a large classroom that facilitated mobility, but it can be adapted for smaller classrooms by splitting the activities between a lecture and lab setting. The interactive learning cycle can be done in a lecture hall, with students approximating based on memory and prior experiences while in discussion with their peers. In that case, the third problem would be changed slightly, perhaps to determine the number of pieces of paper that would be needed to completely cover the classroom walls. The students could discuss linking the size of the paper to other things they could measure from their seat (ceiling tiles or chairs, for example). The lab activity could remain largely unchanged, with the expectation that the students would leave and return to the lab classroom.

For teachers who prefer to avoid having students measure physical dimensions of objects, the estimation problems could be changed to ones that are more abstract, such as the amount of water that would be needed at a day-long music festival. Likewise, for students with more science background, estimation problems could build upon that knowledge; for example, how many of a particular kind of cell could fit on a penny's surface. The purpose of the learning cycle remains the same: to focus on being explicit with assumptions and how measurements are taken, how units are converted, and how to clearly answer the question that is asked, supporting that answer with evidence. These skills are then practiced in the laboratory activity.

Future lessons in a course could continue to build upon this activity, especially if a learning outcome for the course is developing skill in estimations. As mentioned previously, some courses choose to integrate estimation problems throughout the entire course; this activity would provide a strong starting point for meeting those outcomes.

This activity also dovetails nicely with other laboratory-type activities where students are expected to measure and perform unit conversions. For example, lessons on statistical and systematic error or uncertainty measurements could naturally follow from this lesson. The results from this activity could be used to introduce those concepts: given a sufficient sample size, the students could plot the class's estimates and compute mean and standard deviation, and they could begin to discuss measures of error analysis.

SUPPORTING MATERIALS

- S1. Good enough estimations – Lecture slides
- S2. Good enough estimations – Group roles
- S3. Good enough estimations – Responses to ILC estimates
- S4. Good enough estimations – Lab template
- S5. Good enough estimations – Responses to lab problems
- S6. Good enough estimations – Lab follow-up assignment
- S7. Good enough estimations – Sample quiz problems

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Table 1. Lesson timeline. The lesson requires approximately five hours of class time, which may be broken down into two individual sessions of two hours apiece, plus approximately a half hour for students to answer post-lab questions and complete their assessment activities.

Activity	Description	Estimated Time	Notes
Interactive Learning Cycle			
Pretest Question	Students respond individually on paper to an estimation question.	5 minutes	• Collect and do not discuss. • Do not allow students to talk or to use the Internet.
Groups and Group Roles Assignment	Organize class into small groups (~4 students per group).	15 minutes	• Provide instruction regarding group roles (Supporting File S2).
Interactive Problems	1. Students answer an estimation problem in groups, writing on a whiteboard. 2. Student groups present a solution to the problem. 3. The instructor leads a discussion focusing on key ideas brought up by that problem. 4. Repeat steps 1–3 for three problems.	90 minutes	• Examples included in Supporting File S1. • Students should be given no more than 10 minutes to provide an estimate. • Students report their answers onto the spreadsheet shared with the class (Supporting File S3). • Focus discussions on the learning objectives: for example, listing assumptions, converting units, or communicating the problem solutions. • If possible, select for presentation solutions that reinforce both creativity and big ideas for discussion. • Each discussion period should require less time, as students become more comfortable with making estimates.
Laboratory Activity			
Estimations Lab	Students complete three estimation questions with their group.	90 minutes	• Lab activity template and follow-up questions included as Supporting Files S4 and S6. • Students report answers into spreadsheet that is shared with class (Supporting File S5). • Circulate among the groups, listening to their discussions and asking questions such as, "How are you going to estimate the land area of campus?" • Encourage students to compare their results with the results of other groups on the class's spreadsheet.
Post-Lab Debrief	Instructor invites students to present their solutions for discussion.	15 minutes	• Highlight and emphasize explicit assumptions, unit conversions, and communication. Praise students for progress and suggest improvements.
Group Discussion Time	Students are given time to refine their solutions in their small groups.	20 minutes	• This time is important for students to modify their initial efforts, especially if they realized that those did not have explicit work where such was needed. • Students can also discuss the follow-up questions with each other. • Upon completion, the presenter is expected to submit the finished lab to the instructor.
Post-Lab Assessments			
Posttest Question	Students respond individually on paper to an estimation question.	5 minutes	• Collect and briefly discuss what would be necessary for a complete solution in preparation for the exam.
Free-Response Questions	After the lab, students respond individually to questions that promote meta-cognition as well as challenge them to build on their group's estimations.	25 minutes	• See Supporting File S6. • Students should be free to discuss the questions with each other but should solve them on their own as practice for the exam. • This may be completed as homework.