

**STUDENT VERSION**

**Risk of Infection?**

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**Abstract**: This project is designed to examine differences between the exponential and logistic growth models in biology and how to apply these models in solving epidemic questions and comparing to actual disease data sets. This project was designed for an introductory section in Calculus II or a course involving ordinary differential equations, which are appropriate for second year undergraduate students majoring in mathematics, physics, and biomathematics. This project provides an opportunity for students to develop their understanding of differential equations and increase their appreciation of mathematics as it applies to solving and modeling a problem of biology.

**SCENARIO DESCRIPTION**

**Part I**

He immediately noticed that most people were wearing a face mask. There were big signs everywhere stating: “Stay Away from Camels and Be Aware of MERS”.

James had never heard of this before, so he decided to do some research while he was waiting for his flight. Using his smartphone, he easily found an article about MERS on the CNN Website. He found it is a virus and MERS stands for Middle Eastern Respiratory Syndrome. He read: “Starting from May 20th, 2015, there was breaking news in South Korea about Middle East Respiratory Syndrome, which was originally discovered in Saudi Arabia (hence the name). On June 7th, 2015, South Korea reported its fifth death from MERS, bringing the total number of confirmed cases to 64. 1,820 people remain quarantined, either at home or in health facilities. And more than 1,300 schools remain closed.”(1) James had also seen Ebola in the news a few years ago, which was really deadly to humans too. From times.com, James found out that MERS doesn’t appear to be able to spread as quickly as Ebola can (2). While Ebola spreads through direct contact with the bodily fluids of an infected person, MERS doesn’t spread easily from person to person, and though it can spread through the respiratory tract, very close contact is needed.

James recalled that there was an outbreak of flu on the campus of Vanderbilt University in the winter of 2013. The campus had to close for three days. And lately, Ebola has been in the news. So, what’s the difference between MERS, Ebola and the flu anyway? So he went back to his notes and constructed a table to show the major traits for each virus (Figure 3). He began to wonder whether a college student in the U.S. was at risk for getting Ebola. Since he was just in South Korea, another worry was whether he might have been exposed to MERS. “And what does disease “spreading” really mean?”

Back at Vanderbilt University, James went to his virology instructor, Dr. McCarroll, for explanations. Dr. McCarroll told him that there are no available vaccines to control either Ebola or MERS, but there was a vaccine to control the spread of flu. It’s made available after the first reported outbreak of the year. As a result, the prevalence of Ebola and MERS could continue to grow over time, while the flu virus would eventually reach a maximum point and become stable. This is why the people who get Ebola and MERS need to be immediately isolated for medical treatment.

James found a very informative website from the New York Times: <http://www.nytimes.com/interactive/2014/07/31/world/africa/ebola-virus-outbreak-qa.html#origination> . As James read about Ebola, he was surprised to learn that it is a zoonotic disease. This means it is transmitted across species. Ebola was discovered in 1976 and was once thought to originate in gorillas, because human outbreaks began after people ate gorilla meat. Scientists now believe that bats are the natural reservoir for the virus, and that apes and humans catch it from eating food that bats have drooled or defecated on, or by coming in contact with surfaces covered in infected bat droppings and then touching their eyes or mouths. The current outbreak seems to have started in a village near Guéckédou, Guinea, where bat hunting is common, according to Doctors without Borders.

How Does the Disease Progress? Symptoms usually begin about eight to 10 days after exposure to the virus, but can appear as late as 21 days after exposure, according to the C.D.C. At first, it seems much like the flu: a headache, fever and aches and pains. Sometimes there is also a rash. Diarrhea and vomiting follow. Then, in about half of the cases, Ebola takes a severe turn, causing victims to hemorrhage. They may vomit blood or pass it in urine, or bleed under the skin or from their eyes or mouths. But bleeding is not usually what kills patients. Rather, blood vessels deep in the body begin leaking fluid, causing blood pressure to plummet so low that the heart, kidneys, liver and other organs begin to fail.

**Questions:**

1. Using external resources to explain the difference between infection, transmission and virulence.
2. Using external sources, describe what positive or negative-sense ssRNA means with regards to virus replication? How does having positive or negative ssRNA make a difference with respect to infection, transmission, or virulence?
3. How can Ebola be transmitted? What makes a virus an “airborne” virus?
4. What kinds of symptoms do flu victims suffer than Ebola victims generally do not?

**Part II**

Dr. McCarroll reminded him that she had discussed two growth models in her virology class that were based on differential equations. Since James was in ordinary differential equations, he decided to do a little reading ahead on the exponential growth and logistics growth models. He learned that these could be used to predict the changes of the number of infected people if the spreading rate of the viruses were provided.

The exponential growth model is defined as dN/dt=rN and the logistic growth model is dN/dt=rN(1-N/K), where N is the number of infected people with this virus, r is the per capita infection rate for people who are exposed to this virus, and K is the maximum number of people in the population that could be infected by this virus, (i.e., the carrying capacity of the population in this model). The exponential growth model can be used to measure the population that continues to grow forever, and the logistic growth model can be used to measure the population that will reach a maximum (carrying capacity) after some time.

**Questions:**

1. Solve and graph the solution of the exponential growth model with the fixed per capita growth rate r = 0.1 and the initial number of infected people: N(0) = 10.
2. Solve and graph the solution of the logistic growth model with the fixed per capita growth rate r = 0.4 and the initial number of infected people: N(0) =1, and the carrying capacity K = 5000.
3. What mathematical principles describe the spread of disease?

**Part III**

After solving the model differential equations, James wondered how the model equations could compare to the real data from different countries. Since Dr. McCarroll had been so helpful, James reached out again to discuss with Dr. McCarroll something called the $R^{2}$ value which James vaguely recalled from statistics class. James reviewed that the $R^{2}$ value is a statistical measure that indicates how closely a model fits collected data. James also found the appropriate real data set that he can compare to the models above: ​​<https://www.kaggle.com/datasets/imdevskp/ebola-outbreak-20142016-complete-dataset?resource=download> .

Luckily James found a free online calculator at <https://www.desmos.com> that calculates the $R^{2}$ value according to (a) the data set entered by the user and (b) the model equation with unknown parameters. The Desmos calculator provides (c) the values of the unknown parameters which give the best possible $R^{2}$ value. James studied the Desmos video tutorial on $R^{2}$ and he realized that his unknown parameters for the logistic model are r and K although the tutorial video has unknown parameters m,b, or a,b,c. <https://help.desmos.com/hc/en-us/articles/4406972958733-Regressions>

James took detailed notes on his process, which he recorded as follows in his notebook.

1. In order to enter the data set into Desmos, use the data downloaded from Kaggle but only use data rows corresponding to one single select country. This country’s data is plotted in Desmos by clicking “+” then “table” then entering the time values into the $x\_{1} $ column and the number of disease cases into the $y\_{1} $column.
2. In order to enter the logistic model equation, use the solution to the logistic growth model with unknown parameters r and K, and with specific initial value N(0) according to the spreadsheet data for your selected country. Remember to use the symbol ~ explained in the tutorial video, rather than an equal sign =.
3. The Desmos calculator will provide the best fit r and K values.

**Questions:**

1. Using external resources explain how $R^{2}$ is calculated, and also the meaning of the $R^{2}$ value in your own words. What is the best value of $R^{2}$ ?
2. Download the data from Kaggle and select only the data rows corresponding to cases in one specific country. State which country you chose, and state the initial value N(0) for this country. Use your initial value to solve for C in terms of r and K, using the general solution of the logistic model.
3. Using James’s notes above, find what values of r and K best fit the data for your selected country using the logistic model.
4. Repeat questions (8) - (10) for one or two additional countries.
5. How do your results compare for the different countries?
6. What new ideas can you suggest to further improve results in modeling this data?

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