

Worsheet Part 2  
Module: Separation of Variables  
Ebola Outbreak in West Africa

### Modeling an Epidemic

If we solve the differential logistic equation:  $dN/dt = rN(1-N/k)$

We get:  $N(t) = (k / (1 + cke^{-rt}))$  (see hw video and hw sheet)

Using data given we can solve for  $c$  and if we use the  $r$  (proportional constant) from the logistic model in part 1 we only need values for  $c$  and  $k$ . We can find  $c$  as we did in a similar manner that we found  $c$  in the exponential model. We can arrive at a value for  $k$  if we take the second derivative and see where the inflection points would be. These would be given in terms of  $k$ . Using the inflection point of the data we can conclude that we can use 28766 for the value of  $k$  (the carrying capacity). We use these values of the parameters in the question below

#### 4. A new model to consider

In a January 2016 news release, the World Health Organization officially declared the Ebola outbreak over for all three countries [WHO]. The number of Ebola cases totaled 28,602 at the end of the epidemic.

Because we have the data for the actual epidemic, we now know that the predictions by the CDC vastly overestimated the impact of the outbreak. Based on the data we have through  $t = 9$ , and the estimated parameters from above, the following logistic or limited growth model can be developed:

$$N(t) = 2876 / (1 + 586e^{-.7924t})$$

Using the following table:

| Date       | $t$<br>(months) | Total<br>Cases |
|------------|-----------------|----------------|
| 2/15/2015  | 11              | 23,218         |
| 3/15/2015  | 12              | 24,666         |
| 4/12/2015  | 13              | 25,791         |
| 5/10/2015  | 14              | 26,724         |
| 6/14/2015  | 15              | 27,305         |
| 7/12/2015  | 16              | 27,642         |
| 8/16/2015  | 17              | 27,952         |
| 9/13/2015  | 18              | 28,220         |
| 10/11/2015 | 19              | 28,454         |
| 11/15/2015 | 20              | 28,598         |
| 12/13/2015 | 21              | 28,604         |
| 1/17/2016  | 22              | 28,602         |

TABLE 2: Ebola cases reported by WHO and published on Wikipedia.

where  $N(t)$  is the total number of cases at time  $t$  months.

After 7 months: Model estimate: \_\_\_\_\_ Actual number of cases: \_\_\_\_\_

After 10 months: Model estimate: \_\_\_\_\_ Actual number of cases: \_\_\_\_\_

After 15 months: Model estimate: \_\_\_\_\_ Actual number of cases: \_\_\_\_\_

After 20 months: Model estimate: \_\_\_\_\_ Actual number of cases: \_\_\_\_\_

b. Remark on whether you expect this model be accurate for the duration of the epidemic.

5. Graph your logistic model and your exponential model and the data from the World Health Organization on the same set of axis for  $1 < t < 24$  months. Compare.

Consider the impact of the carrying capacity on the accuracy of the logistic model and address whether it is reasonable to develop and use this model during an outbreak, in particular early on before the inflection point appears in the data.