Literature Review: Inclusive Calculus for Life Sciences

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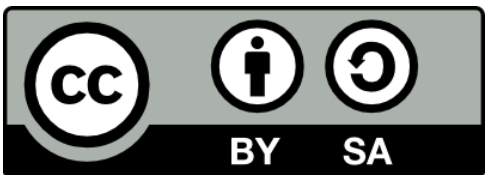

Preamble

Calculus has an image problem in the biology major, and students are anxious about taking it due to self-efficacy. Calculus, as a major gateway course, is the source of a retention problem in biology and in particular, is the source of ever-widening achievement gaps for marginalized individuals in the STEM pipeline. In STEM, life science students are often the most math-averse and bring previous trauma and related math anxiety into the calculus classroom. Biology educators complain that they do not see improved problem-solving skills in students who have taken calculus. In addition, biology is becoming more dependent on data and computation skills rather than the proof-based and analytical methods common in traditional calculus.

We believe that a biology-relevant calculus experience (biocalculus) can help overcome many of these issues in calculus. While challenging to build the capacity for teaching courses, students in biocalculus articulate more detailed connections to the theory of their disciplines and find it more relevant. Biocalculus leverages students' passion and knowledge about biology, and students who enroll in biocalculus courses have greater learning gains than students in traditional calculus courses and lower fail and drop rates. Finally, as biocalculus moves to be more relevant in an age of big data and computing, we explore the epistemological consequences of this shift in the last decade.

In this annotated bibliography, we have gathered resources to expose these issues, inform a more inclusive calculus, and skills in calculus that life science majors will use, and explore the epistemological barriers that keep calculus classes locked into the existing syllabus and practices.

About the Authors

Harriet:
I am a double major in Biochemistry and Japanese Language. During college, my relationship with calculus fluctuated. I was not confident in my skills and felt unwelcome in that class environment. If I were to define myself as a mathematician, it would be that I engage with everyday mathematics, but I do not take math courses in pursuit of a career. Because of the known dread that I and other people of color face walking into that calculus classroom, I focused my time on how Calculus is exclusionary and how some colleges include equity and inclusive teaching into their curriculum.

Naziea:
I am a Neuroscience major minoring in Africana. I have struggled with my quantitative abilities throughout my academic career, notably, calculus was very difficult for me. I have found that engaging with interdisciplinary approaches to applying mathematics has aided me the most when it comes to applying my quantitative skills.

Aleisha:
I am a double major in Biochemistry and Politics. I am also an international student from Mexico City, Mexico. I grew up really liking math and being known for fully engaging in it. In college, my relationship with mathematics changed. Calculus I and II are the lowest grades on my transcript. Because of how drastic my experience was from going to study math back at home to now in the U.S., I studied the epistemological shift in calculus teaching throughout the last 10-20 years.

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1. Biocalculus

1.1 Bennoun 2022

Citation: Bennoun, S. (2022). Transforming a Calculus for Life Sciences Course: Moving From Procedural Calculus to Studying Dynamical Systems and Bifurcation Theory. *PRIMUS*, *32*(2), 246–259. https://doi.org/10.1080/10511970.2020.1827104

Summary: There has been an epistemological shift in biocalculus to focus on systems ideas instead of "procedural calculus." This article explores that. Their suggested methodology:

- Embrace active learning
- Use teaching materials to facilitate active learning
- Uses bifurcation theory to explore the application of mathematics in life sciences

1.2 Bermuli and Tamba 2021

Citation: Bermuli, J. E., and K. P. Tamba. "Is Project-Based Transdisciplinary Assessment Effective in Reducing the Mathematical Anxiety of Pre-Service Biology Teacher?". *Biosfer: Jurnal Pendidikan Biologi*, Vol. 14, no. 2, Oct. 2021, pp. 238-49, https://doi.org/10.21009/biosferjpb.20556

Summary: Investigates how project-based transdisciplinary assessment can or cannot help with math anxiety. showed no significant difference in mathematics anxiety before and after the implementation of project-based transdisciplinary assessment. This was indicated by non-significant z-values (e.g., $z = -0.163$) and p-values (e.g., $p = 0.87$). They suggest the need for continuous, collaborative, and institutional efforts in transdisciplinary applications to improve the quantitative abilities of pre-service Biology Teachers.

1.3 Bodine et al 2020

Citation: Bodine, E.N., Panoff, R.M., Voit, E.O. et al. Agent-Based Modeling and Simulation in Mathematics and Biology Education. Bull Math Biol 82, 101 (2020). https://doi.org/10.1007/s11538-020-00778-z

Summary: Advocate for the inclusion of ABMs in undergraduate biology and life science curricula, consistent with the recommendations of the Bio2010 Report. They suggest that all students should have some exposure to mathematical modeling, including ABMs. While not every student may need a course exclusively focused on ABM, they should have the opportunity to utilize ABMs to perform experiments, collect data, and analyze results. In this paper, there was no assessment on a specific demographic.

Relevant quote:
" Biological applications are ideal systems for first attempts at agent-based models as they typically do not necessitate learning extensive new vocabulary and theory to understand the basic components that need to be included in the model."

1.4 Caughman and Weigel 2022

Citation: Caughman, Alicia M., and Emily G. Weigel. "Biology Students' Math and Computer Science Task Values Are Closely Linked." *CBE—Life Sciences Education* 21, no. 3 (2022): ar43. https://doi.org/10.1187/cbe.21-07-0180.

Summary: This study compares math and computer science, and discusses demographic assessment looking into students' perception of math vs computer science. Researchers found that biology students had more of a positive outlook toward computer science. They also found that gender did play a role in students' perception of computer science. Based on their findings, they recommend introducing CS and math early and integrating them into the biology curriculum. To encourage students to view computer science as more useful than it is seen as challenging.

Relevant quotes:
"Positive correlations between students' attitudes about math and CS indicate that individual students tend to view both subjects similarly. This correlation is strongest for utility, followed by interest, then cost."

"Biology departments may help students to take and see relevance in math and CS by offering at least one combined math-CS–based biology course, rather than relying on student skill development through course work offered separately through math and CS departments."

1.5 Durán and Marshall 2019

Citation: Durán, P. A., & Marshall, J. A. (2019). Mathematics for biological sciences undergraduates: a needs assessment. *International Journal of Mathematical Education in Science and Technology*, *50*(6), 807–824. https://doi.org/10.1080/0020739X.2018.1537451

Summary: This study examines the quantitative needs of biology students, emphasizing the importance of considering disciplinary variations.

1.6 Hausberger and Godelle 2023

Citation: Hausberger, T., & Godelle, B. (2023). Meeting the biocalculus challenges: a reflection on didactic transposition processes in a cross-disciplinary context. *The Learning and Teaching of Calculus Across Disciplines*, MatRIC, Bergen (NO), Norway. pp.109-112. https://hal.science/hal-04239935/

Summary:
They examine how university-level math courses adapt over time, they examine a biocalculus course to over 700 first-year biology students with the help of 10 mathematics instructors. Made by an evolutionary anthropologist who works in quantitative biology. Using ATD theoretical concepts to answer questions such as: What conditions allowed the emergence of this biocalculus course at the University of Montpellier (UM, France)? How does it differ from a traditional calculus course?

Relevant quotes:
"Despite various institutional injunctions since the early 2000s in the USA, the desired reform of a greater integration of mathematics and biology in the undergraduate curricula seems slow to materialize."

"Verhulst logistic model, etc.) which are conducive to articulating mathematical work with biological stakes that meet the combined requirements of mathematics and biology teaching institutions, and which can be found in the biocalculus textbooks."

1.7 Luque et al 2022

Citation: Luque, A., Mullinix, J., Anderson, M., Williams, K. S., & Bowers, J. (2022). Aligning Calculus with Life Sciences Disciplines: The Argument for Integrating Statistical Reasoning. *PRIMUS*, *32*(2), 199–217. https://doi.org/10.1080/10511970.2021.1881847

Summary:
An urban university in the southwestern USA decided to revamp a course to better equip students with statistical and data modeling skills needed in biological sciences. The researchers integrated data analysis and showed students how math applies to their future careers. Over four semesters, the course saw a doubling in enrollment, a more than 50% reduction in failure rates, and positive student responses to using math in biology. The course redesign led to increased enrollment, decreased non-passing grades, and positive student attitudes toward mathematics.

Relevant quotes:
"The non-passing grade rate in the redesigned course was on the 10–20% range, which is much lower than the non-passing grade prior to the redesign (48 ± 15%), and the student attitudinal response indicated a positive perception of the course, in particular regarding application and real-world value."
"The student attitude towards thinking versus "plug and chug" was more positive, and the recognition of applications and real value of the course received the most positive responses."

"Based on the performance of students in exams, homework, and lab activities, our assessment is that, on average, students demonstrated a good understanding and application of basic data analysis and calculus concepts."

"Our emphasis on integrating biology and social examples that are familiar and compelling contributed to students' perception of the value of mathematics."

"This course redesign using the CRAFTY recommendations led to an increase in enrollment and a positive response towards the application and real value of Calculus for Life Sciences."

1.8 Marshall and Durán 2018

Citation: Marshall, J. A., & Durán, P. (2018). Are biologists getting the mathematical training they need in college?. *Biochemistry and molecular biology education: a bimonthly publication of the International Union of Biochemistry and Molecular Biology*, *46*(6), 612–618. https://doi.org/10.1002/bmb.21178

Summary: Several major scientific organizations recommend a revisit of the calculus curriculum. This paper specifically focuses on revisits that should be made to the realm of calculus applied in a biological context. Large surveys were conducted to a group of research experts to examine their knowledge, training, and use of mathematics and statistics in their everyday work. The results of the surveys indicated that biologists needed a base in descriptive and inferential statistics and mathematical modeling, including

some concepts from calculus, focused on conceptual understanding rather than computational accuracy, followed by differentiated programs of study in mathematics and statistics, depending on the intended discipline within biology.

1.9 Jungck, Robeva, and Gross 2020

Citation: Jungck, J.R., Robeva, R. & Gross, L.J. Mathematical Biology Education: Changes, Communities, Connections, and Challenges. Bull Math Biol 82, 117 (2020). https://doi.org/10.1007/s11538-020-00793-0

Summary: This article discusses the intersection of biology and math and some recent findings that promote the inclusion of math and biology. It touches on the importance of holistic learning but only references other studies.

Relevant quotes:
"Finally, in "Mathematical Biology: Expand, Expose, and Educate!" Lee and Clinedinst (2020) raise one of the hardest, most fundamental challenges in undergraduate STEM education: how do we redress the systemic exclusion of historically underrepresented groups in mathematics, biology, and interdisciplinary programs that involve both."

"In "Undergraduate Quantitative Biology Impact on Biology Preservice Teachers," Mayes et al. (2020) stress the need to provide more opportunities in the classroom for authentic student experiences that demonstrate why quantitative reasoning (QR) is an essential tool for learning biology."

2. Quantum Skills for Biologists

2.1 Garfinkel et al. 2022

Citation: Garfinkel, A., Bennoun, S., Deeds, E. *et al.* Teaching Dynamics to Biology Undergraduates: the UCLA Experience. *Bull Math Biol* **84**, 43 (2022). https://doi.org/10.1007/s11538-022-00999-4

Summary: Discusses how a modeling course was implemented due to students interested in life sciences deeming calculus unapproachable. They made a new course where they teach students to model biological relationships as systems that change over time. They show these changes using arrows called "change vectors." This new course, taken by about 1400 students per year, has made students like math more, reduced differences in performance between different groups of students, and helped students do better in physics and chemistry. The paper also goes more in-depth and discusses what they taught to the students, for instance, they explain how they taught linear algebra in the second portion of the class, explaining the concept of linear functions. The authors think this approach can work in other schools too, no matter their size or the types of students they have. They also want to remind everyone that all the materials they used to teach this course are free for anyone to use.

Relevant quotes:

"Initial surveys of life science faculty revealed that many felt that the standard math department offering lacked biologically relevant examples and did not include methods that have become critical in math for biology, such as mathematical modeling and computer simulation."

"When students who had followed the LS30 series in Fall 2014 and Winter 2015 were surveyed, 94.2% somewhat or strongly agreed with the statement "I saw the real-life application or relevance of what I learned." When asked about whether the course had increased their confidence in their mathematics ability, 79.7% of the respondents somewhat or strongly agreed. Similarly, 74.8% declared that LS30 helped increase their science ability. Finally, 90.8% of the respondents declared that they were looking forward to taking more life science courses."

"for students who took LS30, the grade differences for underrepresented minority students or first-generation students were smaller, and not statistically significant."

"By contrast, many students who had followed at least a part of the traditional Calculus for Life Sciences course (Math 3) saw their confidence in their math ability decline."

2.2 Norton 2022

Citation: Norton, D. E. (2022). A Course on Mathematical Modeling for the Life Sciences. *PRIMUS*, *32*(2), 154–167. https://doi.org/10.1080/10511970.2021.1881848

Summary: This article describes a mathematical modeling course. Although this is considered to be a course separate from biocalculus, many of the topics overlap with other incarnations of biocalculus which include discrete approaches.

2.3 Stoner and Joyner 2022

Citation: Stoner, M. A., & Joyner, R. L. (2022). Breathing Life Into Calculus: Using Simulation to Enhance Students' Understanding of the Relevance of Calculus to Physiology. *PRIMUS*, *32*(2), 260–277. https://doi.org/10.1080/10511970.2020.1827102

Summary: This article focuses on a way to add hands-on simulation to a modeling experience in the calculus classroom. This describes an activity that can be used in any calculus class - not just biocalculus.

2.4 Schumacher and Siegel 2015

Citation: 2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences Carol S. Schumacher and Martha J. Siegel, Co-Chairs Paul Zorn, Editor

Summary: Overall, they discuss the importance of calculus and the different ways in which calculus can be implemented in a way that increases the positive perception of calculus. As well as increase student success in calculus. They also discuss the trends that they have seen, for instance, biology students' increased enrollment in the course. A lot of students are enrolling into calc and fail at calc and many students, even if they do succeed, do not continue using their skills from calc. More biology students are

enrolling in calc I instead of math and engineering majors. Many universities are trying to make the course more challenging for those who took the AP course in high school.

Relevant quotes: "CUPM is troubled both by the level of students' pre-calculus preparation and by the Introduction permanent exodus from mathematics of many good students who receive college credit for high school or AP calculus or are discouraged by their experience in college calculus classes." Students do fair worse in Calc II and III. Research is being done to further student success.

"Calculus is the foundation of the study of continuous processes, and thus a gateway to the physical, biological, and social sciences. Instruction in calculus should include consideration of both single- and multivariable approaches."

"Combining ideas from linear algebra and calculus helps students develop facility with visualization, see connections among mathematical areas, and appreciate the power of abstract thinking."

"George Thomas produced the first edition of his Calculus and Analytic Geometry in 1951. In the late 1980s and early '90s, the Calculus Reform Movement produced several curricula that front-loaded material on differential equations. This came from a recognition that Calculus is about the study and modeling of dynamical systems."

"The emphasis within Calculus has traditionally been on derivatives as slopes of tangent lines and integrals as areas—a very static interpretation that makes it difficult for many students to transfer these tools to dynamical situations. Current work on calculus recognizes the central importance of the concept of covariation, understanding how change in one of two or more linked variables is reflected in change to the other variable or variables [5]."

"it is more useful to see the derivative as the instantaneous rate of change or as a measure of sensitivity of change in one variable to change in another, rather than emphasizing its role as a method of finding slopes. Instead of privileging the integral as area, the emphasis should be on measuring accumulation." Students better grasp the material this way.

"FTC as stating the equivalence of two ways of understanding the definite integral: as the change in the value of an antiderivative or as the limit of a summation." Better than understanding integration as antidifferentiation, students don't grasp that.

"Several colleges and universities have moved sequences and series out of Calculus II and replaced them with an introduction to multivariable calculus."

Lessons from the MAA Study of Calculus I

Discussing monitoring successful calculus programs and student outcomes. Discusses how they went about monitoring the programs.

Discusses making sure that students get the assistance they need in order to succeed in the class.

Resulting in the students who, in the past, were well served by the traditional calculus course no longer studying calculus in college. They jump directly to more advanced courses. The primary audience for Calculus I has shifted to biology majors and others for whom the appropriate emphasis is the investigation of dynamical systems with much greater reliance on computation. We know more about how students learn and about the importance of creating an active learning environment in which students are forced to wrestle with mathematics and build a robust personal understanding. The availability of online resources provides opportunities for instructors to spend less time lecturing and more time in active learning, directly engaging students with the critical concepts of calculus. With greater attention to the institutional bottom line, there is less tolerance of high failure rates as well as of students who go on without the knowledge or skills needed to succeed in subsequent courses. Placement, support, and effective instruction are more important than ever. Many very bright and talented mathematicians and mathematics educators already realize these points and are developing a variety of tools, curricula, and pedagogical approaches to meet the needs these courses serve. At the same time, we are getting better at assessing the effectiveness of these innovations. We now have a much clearer grasp of what works and why it works. There is support for reform of undergraduate mathematics instruction coming from influential actors: the White House Office of Science and Technology Policy, the National Academies, and the Association of American Universities among many others. The American Mathematical Society has joined MAA in recognizing the need for more effective undergraduate instruction, and a variety of joint efforts are now underway.

Prerequisites for calculus

Basic proficiency in algebra, combined with some analytical maturity, is all that is required. Some data analysis courses could have calculus as a prerequisite.

To further retention of the material, quote: “Students should be encouraged to take an applied data analysis course at any time in the first two years, which could be before, concurrent with, or after calculus.” pg. 41

(then discusses prereqs, calc being one for future courses)

Sample Syllabus for Statistics pg. 43

Undergraduate Preparation for PhD Programs in Mathematics

Discusses how fundamental an understanding of calculus is for a PhD. For recruitment, these courses should be given to students when best appropriate for their “mathematical maturity”

Calculus and Precalculus

Discusses wanting students to succeed, etc. pg. 78

The rest of the paper discusses ways in which faculty can encourage better outcomes in calculus and later discusses faculty issues before concluding.

Exploration

Quote: "The goal of exploration is for students to engage with examples, deepen understanding, notice patterns, and make conjectures, thus providing a natural setting for recognizing the importance of proof to support correct conjectures. To be effective, exploration must culminate in clear statements of what has been discovered. Teachers must follow up and be sure students understand what the exploration has achieved." pg. 91

2.5 Spencer et al 2023

Citation: Spencer, D., Willis, C., Shen, Y., Fenn, M., & Viel, S. (2023). Utilizing a Blended + Flipped Learning Approach in a Calculus for Life and Management Sciences Classroom. *PRIMUS*, *33*(6), 670–690. https://doi.org/10.1080/10511970.2022.2122645

Abstract: This paper described a "flipped" classroom approach - moving instruction to homework and the homework/doing/applying to the classroom. At the same time, class size was reduced to promote active learning. Students perceived this change positively. In addition, instructors link to relevant resources for making similar changes as your institution.

2.6 Thompson et al 2023

Citation: Thompson, K.V., Nelson, K.C., Sniezek, J. et al. Online, Interactive Modules Improve Quantitative Skills in Community College Biology Students. J Sci Educ Technol 32, 824–836 (2023). https://doi.org/10.1007/s10956-022-10020-1

Summary: This paper evaluates the effectiveness of MathBench Biology Modules in enhancing the quantitative skills and attitudes toward integrating math and biology among students in an introductory biology course at a community college. MathBench Biology Modules were developed to address this gap by leveraging the advantages of online learning, such as self-pacing, practice opportunities, and immediate feedback. Biology courses, they believe, tend not to prepare students for future quantitative biology research.

Relevant quotes:
"The MathBench group showed a significant improvement in quantitative skill test scores across the semester, while the comparison group showed no improvement (time*group interaction: $F1,142 = 5.4389$, $p = 0.0211$, Fig. 1)"

"Students who were concurrently enrolled in a math course along with the Introductory Biology course showed greater improvements than those who were not concurrently enrolled in math (time*concurrent enrollment interaction: $F1,142 = 8.1788$, $p = 0.0049$)."

2.7 Walker et al 2023

Citation: Walker, B.J., Townsend, A.K., Chudasama, A.K. *et al.* VisualPDE: Rapid Interactive Simulations of Partial Differential Equations. *Bull Math Biol* **85**, 113 (2023). https://doi.org/10.1007/s11538-023-01218-4

Summary: Discusses a web-based tool called VisualPDE that can solve a wide range of time-dependent Partial Differential Equations (PDEs) in real-time. They go into the background of the tool, how it was

designed, and how it can be used, and they give examples of how it's been used in teaching, research, and sharing knowledge. For example, with research they can use this tool to examine spatial models. This tool can be used with teaching materials to make class materials more accessible. They hope VisualPDE will be useful for many different groups of people who use math and that it encourages connections between different fields through easy-to-use computing. They also suggest that in the future, research papers in various fields could include links to interactive simulations like the ones VisualPDE uses. They advocate to keep improving VisualPDE and to see how it might change the way we interact with and talk about STEM. Hopefully, VisualPDE will encourage more efforts to make math interactive, which they think is an important and exciting way to advance how we understand and share science.

Relevant quote:
"VisualPDE proves to be useful across many mathematical communities, and that it facilitates and inspires interdisciplinary connections through interactive, accessible, shareable computing."

"One advantage to web-based tools such as VisualPDE, GeoGebra and Desmos is that they can be natively embedded in such webpages, providing seamless connection with the material being described."

3. Calculus Epistemology

3.1 Aikens 2020

Citation: Aikens, M.L. Meeting the Needs of A Changing Landscape: Advances and Challenges in Undergraduate Biology Education. *Bull Math Biol* **82**, 60 (2020).
https://doi.org/10.1007/s11538-020-00739-6

Summary: National reports on undergraduate biology education and a growing discipline-based education research community in biology have fueled significant changes over the last 25 years in undergraduate biology classrooms. Inquiry-based and discovery-based methods have been incorporated into laboratories, allowing students to *do* science in order to foster their understanding of the nature of science. Data have demonstrated the efficacy of active learning methods, resulting in increased usage in a wide variety of biology classrooms. Programs and teaching methods aimed at broadening participation of underserved groups in biology have been developed and implemented, though more work needs to be done to enhance the inclusivity of the biological sciences. As biology educators continue to work to improve the learning environment, increasing attention should be focused on integrating biology with other STEM disciplines. Although some innovative interdisciplinary courses and programs have been developed, reform efforts have not been widespread. However, as science research becomes increasingly interdisciplinary, it will be critical for biology graduates to be able to draw upon other STEM disciplines as they tackle the problems of the twenty-first century.

Relevant quotes:
"If biology programs are going to require calculus and physical science courses as major requirements, presumably because there is value in learning calculus and physics for understanding biology, then we owe it to our students to *explicitly demonstrate* the value of these disciplines to biology."

"biology educators should take the initiative to ensure our biology students are learning how mathematics and the physical sciences can be applied to biology."

"Too often, biology majors complain that they do not understand why they have to take required mathematics and physical science courses, particularly calculus and physics, for their biology degree. Students lack an understanding of how biology can be informed by other STEM disciplines because instructors often fail to make explicit connections between biology, mathematics, and the physical sciences."

"These recommendations were built on the knowledge that mathematics and the physical sciences play crucial roles in understanding biological systems, and that concepts and principles from these disciplines can be applied to answer biological questions."

3.2 Alabdulaziz 2021

Citation: Alabdulaziz, M.S. COVID-19 and the use of digital technology in mathematics education. *Educ Inf Technol* **26**, 7609–7633 (2021). https://doi.org/10.1007/s10639-021-10602-3

Summary: Results show different types of digital technology used in mathematics education included (mobile technologies, touchscreens, and pen tablets, digital library and designing learning objects in mathematics education, Massive Open Online Courses (MOOCs) in mathematics, and computer algebra systems (CAS) such as Mathematical, Maple, MuPAD, MathCAD, Derive and Maxima.), and the effects varied by the type of educational technology used. However, in view of the COVID-19 school closure period, it is apparent that digital learning in mathematics education is the instant positive response. COVID-19 changed the way that technology is seen and used in the classroom, including mathematics. This was only done in schools in Saudi Arabia and no other geographical region. (mostly discusses the usage of technology during the pandemic, will most likely not be as useful.)

Relevant quotes:

"The above table shows that 98% of participants believed that COVID-19 is the gateway for digital learning in mathematics education. 97% claimed that the use of online education by schools had expanded greatly following the coronavirus outbreak."

"The participants mentioned that when the student finds it difficult to solve the task in mathematics, he can access to the mobile technologies and open the videos see the solutions, which allows students to learn at their own pace and in their own learning style. The researcher thinks that students and teachers are given new experiences through the application of mobile devices as instruments in mathematical education."

"Nonetheless, the availability of mobile technologies for students forms a relation between students and mathematics that is not broadly embraced by mathematics teachers, which interrupts the conventional transferal of mathematics knowledge between teachers and students, and has received minimal attention in the literature."

3.3 Page et al 2024

Citation: Page, Elizabeth M., Samuel S. P. Shen, and Richard C. J. Somerville. "Can We Do Better at Teaching Mathematics to Undergraduate Atmospheric Science Students?". *Bulletin of the American Meteorological Society* 105.3 (2024): E680-E685. https://doi.org/10.1175/BAMS-D-22-0245.1 Web.

Summary: Explores strategies for improving the teaching of mathematics to undergraduate students in atmospheric science programs. It addresses the challenges students face in learning mathematics within the context of atmospheric science and suggests ways to enhance their understanding and efficiency in this subject. Advocates for a more holistic and interdisciplinary approach to teaching mathematics to undergraduate atmospheric science students, aiming to enhance their mathematical skills and better prepare them for careers in the field.

Relevant quote:
"Why has math education in atmospheric science become what it is? We attribute its shortcomings to at least one conspicuous defect: In the traditionally outsourced mathematics courses, the examples given in mathematics textbooks and the questions posed in their problem sets are, unsurprisingly, not related to atmospheric science or oceanography or climate science. Thus, from a purely pedagogical point of view, these textbooks miss a promising opportunity to show the student how mathematics is applied in the major field."

3.4 Redish and Kuo 2015

Citation: Redish, E.F., Kuo, E. Language of Physics, Language of Math: Disciplinary Culture and Dynamic Epistemology. *Sci & Educ* **24**, 561–590 (2015). https://doi.org/10.1007/s11191-015-9749-7

Summary: In a standard interpretation, lack of success with math in physics is attributed to a failure to transfer mathematical skills from math class to physics class. The considerations raised here suggest an alternative possible diagnosis: Learning math in math class and math in physics class should be treated as learning two related but distinct languages. Although there is significant overlap, there are also important differences, and expertise in one does not guarantee expertise in another. Although both physics and math make meaning in the way that any language does—by building on physical experience, by drawing on broad (encyclopedic) knowledge, and by contextualizing—they do so in different ways. We must recognize that we are asking students to become bilingual and provide instruction and practice in blending in physical interpretation on mathematical formalism. This perspective raises significant issues both for research and for instruction. (this paper is more so focused on how symbols can be difficult for students to understand; solely focusing on physics will probably not be helpful)

Relevant quotes:
"To succeed in physics, students need not just to be fluent with mathematical processing in the context of physics, but also with the mathematical modeling of physical systems, blending physical meaning with mathematical structures, and interpreting and evaluating results."

"To help students integrate physical meaning with the mathematics they are learning and using, we have to use our knowledge of what productive knowledge they possess, what can cue different knowledge

elements, what epistemological resources students tend to use, how they feel about physics and math, and how all these elements interact."

3.5 Stockton 2010

Citation: Stockton, James Clinton. "A Study of the Relationships between Epistemological Beliefs and Self-Regulated Learning among Advanced Placement Calculus Students in the Context of Mathematical Problem Solving," 2010.

Summary: Findings from this study indicate that participants' *unique* and *arbitrary* beliefs regarding problem solutions manifested during think-aloud sessions affect their actions in the definition of the task phase. When faced with a task involving arbitrary constants or variables, students with *unique* beliefs may struggle with the role of the arbitrary constants, whereas students with *arbitrary* beliefs will tend to develop a more appropriate definition of the task. The expectation that such problems should have unique solutions places a barrier between the student and the appropriate understanding of the problem. The belief that problems with arbitrary constants had *unique* solutions led participants through fruitless explorations devoid of purposeful transitions or elicited low-level monitoring of flawed plans for procedural accuracy. In contrast, participants who exhibited *arbitrary* beliefs regarding problem solutions in this study tended to make more purposeful transitions as problem navigation informed the changing nature of the constants. The results of this study indicated that students' *rational* and *empirical* approaches to mathematical problem-solving are related to processing during the performance control phase. Findings from this study indicate that mathematical resources and epistemological beliefs work in tandem in relation to students' SRL processing of problem-solving tasks. All participants passed the prerequisite course, AP Calculus AB, which dealt with topics presented in both the think-aloud and AP Calculus exam preparation problem-solving sessions. Exposure to mathematical content does not imply accessibility (Schoenfeld, 1985, 1992). Epistemological beliefs are indirectly related to the accessibility of mathematical resources. 286

4. Inclusive Teaching in Calculus

4.1 Hagman and Fosdick 2017

Citation: Hagman, J.E., Johnson, E. & Fosdick, B.K. Factors contributing to students and instructors experiencing a lack of time in college calculus. *IJ STEM Ed* 4, 12 (2017). https://doi.org/10.1186/s40594-017-0070-7

Summary: Calculus is known to dissuade students from pursuing any math or life science majors from continuing. There are different reasons for this: the student's identity, lack of experience, and low student-centered teaching. This journal explores the different factors that contribute to the low retention rate of underrepresented groups of people in calculus courses, looking into both the students' and instructors' perspectives. For students, we know that what they experience outside of class in terms of their identity and experiences in class can translate to their learning ability inside the calculus classroom. For instructors, their decisions on the structure of the class are more focused on how they plan to teach and what they want to put emphasis on. Because of the low number of students and instructors in different racial/ethnic groups, people of color were not the main focus of the study; the study focused more on gender-specific problems where they had more people to represent those groups. Estrella Johnson is a

professor in the mathematics department with research focussing on the pedagogical practices of mathematicians and undergraduate mathematics education. Bailey K. Fosdick is an institute data scientist and earned her PhD in developing statistical methods to analyze multiverse data, often focusing on applications in the social and life sciences.

Relevant quotes:
"Many of the students leaving STEM majors cite ineffective teaching methods and uninspiring atmospheres in introductory-level STEM courses—with introductory mathematics courses often singled out—as the primary reason for attrition."

"Together, this indicates that factors stemming from students' backgrounds, their mathematical dispositions, and their classroom experiences contribute to how they experience and actualize content coverage, exposure, and emphasis."

"…students who are women and students who have not taken calculus before are more likely to report negative OTL compared to men and compared to students who took calculus in high school, respectively."

4.2 Hagman 2021

Citation: Hagman, J. E. (2021). The Eighth Characteristic for Successful Calculus Programs: Diversity, Equity, & Inclusion Practices. *PRIMUS*, *31*(1), 70–90. https://doi.org/10.1080/10511970.2019.1629555

Summary: Looking into schools with a higher percentage of calculus students, the research group identified 7 characteristics that make calculus programs successful and later came up with the eighth characteristic that can help all students become successful in a calculus course instead of the usual white or Asian and male demographic. Hagman explained the importance of having the eighth characteristic increase the support and success for marginalized groups in their calculus course and pursuit of STEM. It is meant to bring more diversity as well as permanently altering the formatting of calculus courses to make it more f for groups other than white and Asian men in the long run. Hagman is a math professor. Her research focuses on undergraduate math education. She has studied calculus programs across the US and learned how to support math departments in improving diversity, equity, and inclusion in their schools.

Relevant quotes:
"Diversity, equity, and inclusion practices described here assume an anti-deficit perspective in which the historical lack of participation and success of underrepresented populations in STEM is understood as not being due to their lacking ability or interest, but rather due to systemic barriers that have functioned to exclude these students from being successful in mathematics, and more broadly in STEM."

"…there are factors outside a departments' immediate and short-term control that affect the diversity of the student population within their calculus programs, including systematic structures that prevent students of color from entering college at the same rates as white students."

“It is increasingly clear to the college mathematics community (e.g., the AMS Blog “inclusion/exclusion”) that a key ingredient has been missing, and college calculus programs will not be successful until diversity, equity, and inclusion is explicitly attended to in relation to calculus programs.”

4.3 Moreno and Muller 1999

Citation: Moreno, Susan E., and Chandra Muller. “Success and Diversity: The Transition through First-Year Calculus in the University.” *American Journal of Education*, vol. 108, no. 1, 1999, pp. 30–57. *JSTOR*, http://www.jstor.org/stable/1085634. Accessed 7 Mar. 2024.

Summary: In 1988, UT Austin made a program called the Emerging Scholars Program (ESP) where students of different identities, especially African Americans, LatinX, and women, were welcome in order to enhance their mathematics skills and decide whether they could continue along the path of MSE (Mathematics, Science, and Engineering) majors. Students in the ESP program did way better than their classmates who weren’t part of that program. There was an emphasis on African Americans, LatinX, and women in the research, with European American students added, but they had less of an influence in the results. There are parts of uncertainty in the results being that men score better overall in the ESP group, and the only group of women who scored higher than men in the program were European American students, while African Americans scored a lot lower than all other groups in the program. I think this paper is important because in similar papers, other researchers do not explain the intricacies of error within the marginalized groups chosen and that there will still be room for uncertainty within the test group used. SUSAN E. MOREN was studying for a PhD in sociology at UT Austin. CHANDRA MULLER was an assistant professor in sociology at UT Austin. Both women have interests in education with their individual perspectives on identity and outside engagement.

Relevant quotes:

“Findings suggest that ESP students earn higher calculus grades than non-ESP students and are more likely to enroll in the second semester of calculus.” *(p 30)*

“Among the freshmen entering college in 1994, 35 percent of African Americans, 38 percent of Latinos, and 31 percent of European Americans intended to major in mathematics-related fields (National Science Foundation 1996). In the previous year, 12 percent of the baccalaureate degrees earned by African Americans were in MSE fields, as were 14 percent of those earned by Latinos and 14 percent by European Americans.” *(p 31)*

“African Americans who participated in the program while concurrently enrolled in calculus were two-three times more likely to earn a B or higher in calculus than program nonparticipants, and they were more likely to persist in MSE areas. African Americans and Latinos are less likely than average to stay in quantitative fields after the first year (Hilton and Lee 1988; Massey 1992), although about the same proportion intend to major in these fields as European Americans (National Science Foundation 1996).” *(p 32)*

“Students' perceptions that MSE faculty members isolate themselves is associated negatively with students' persistence in a mathematics-related major, particularly among underrepresented populations (Astin and Sax 1996). African Americans, Latinos, and women who previously enjoyed nurturing

relationships with secondary teachers often view MSE faculty members at the university as unfeeling and insensitive because they tend to use a straight lecture style for instruction and have very little direct interaction with students (Seymour 1995; Seymour and Hewitt 1997)." *(p 33)*

"The ESP was established in the fall of 1988 by the Department of Mathematics and the College of Natural Sciences at UT to increase diversity among mathematics students. Modeled after the Mathematics Workshop Program at the University of California, Berkeley, it was designed to facilitate positive experiences for students in freshman calculus by providing them a diverse, supportive group with whom they could share and exchange calculus knowledge."

"Through a small-group format, the ESP provides students with the opportunity to develop and maintain friendships with others who are enthusiastic about learning calculus and pursuing mathematics-related majors. The ESP administrators do not view it as a minority student program; instead, they focus activity on students learning calculus." *(p 35)*

5. Identity Formation in Math and Biology

5.1 Buckmire et al 2023

Citation: Ron Buckmire, Carrie Diaz Eaton, Joseph Hibdon, Katherine M. Kinnaird, Drew Lewis, Jessica Libertini, Omayra Ortega, Rachel Roca & Andrés R. Vindas Meléndez, "On Definitions of "Mathematician,"' *Journal of Humanistic Mathematics*, Volume 13 Issue 2 (July 2023), pages 8-38. DOI: 10.5642/jhummath.ZRUZ1463.

Summary: This journal explores the complexities surrounding defining a mathematician, asking questions like, can anyone be called a mathematician? Certain definitions could either increase diversity or exclude different groups of people, so different types of definitions are used to describe who a mathematician is and when to use those different definitions. There are function-based, qualification-based, and identity-based ways to define a mathematician, as well as broad and narrow ways. The writers of this journal included their own positionalities, defining how they interpret themselves as mathematicians. We also used their positionality formatting to explain our experiences with math and why we decided to work on this project.

Relevant quotes:
"...all authors of this paper were classified as a "mathematician" under the following seven example definitions: Functional-Broad, Functional-Hybrid, Qualifications-Broad, Qualifications-Hybrid, Identity-External, Identity-Hybrid, and Compuesto-Broad; therefore those columns are not included in Table 1. In these seven common definitions, there are both kernels of what it generally means to be a mathematician and qualities unique to individuals who have been entrenched in, and validated by, this field for many years."

"...when presented with a definition, it is important to interrogate: who is making the definition? In what ways does this choice work towards or against justice? What groups benefit from the choice being made, and who is harmed?"

5.2 Good and Dweck 2012

Citation: Good, C., Rattan, A., & Dweck, C. S. (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. *Journal of Personality and Social Psychology, 102*(4), 700–717. https://doi.org/10.1037/a0026659

Summary: In order to help create and validate women's sense of belonging in math, the idea of mathematics being a fixed trait and the stereotype that women are less capable than men had to be removed. Validating women's sense of belonging, in turn, lowered men's, and that lowering helped women, in turn, want to pursue math more. I think this is a good paper to add because the Math Sense of Belonging Scale they use reminds me of the definition of mathematics paper, where everyone explained what their mathematical identity was.

Relevant quotes:

"They claimed that gender gaps exist, not because of discrimination, but rather because of women's lack of desire to pursue math-based disciplines, such as physics or engineering. This position portrays women's versus men's desire to pursue certain subjects as an inherent part of their natures that lead inevitably to gender disparities in representation."

"…women's assumed lack of desire to pursue math may serve as a readily available explanation for their departure from the field, it is possible that a lack of desire to remain in math may stem from a lowered sense of belonging rather than from a natural disinclination on the part of women."

"[T]he purpose of this research was to study sense of belonging to an academic domain to better understand factors that might contribute to the representation gaps that exist between the sexes and, perhaps by extension, among various ethnic or racial groups."

"Females who, despite the stereotype, find themselves in math-related disciplines must now face the "culture of talent" pervading these fields, a culture that may also undermine their sense of belonging."

5.3 Griffith 2010

Citation: Griffith, A. L. Persistence of women and minorities in STEM field majors: Is it the school that matters? *Economics of Education Review* **29**, 911-922 (2010). https://doi.org/10.1016/j.econedurev.2010.06.010

Summary: This article focuses on educational economics, explaining why there is low persistence in the STEM field for women and minorities. This paper discusses different reasons why women and minorities have a harder time staying in the field, as they are picking out their major and going to classes with different professors and students. Some of the factors were the number of women and minority faculty members, the types of schools that they are going to—such as going to a STEM-specific school vs. liberal arts vs. a school that has graduate students in them—and their support systems. This paper targets the systematic level in which groups of students are not succeeding. Amanda L. Griffith is an Economics professor at Wake Forest University who has an interest in studying the economics of higher education, and because she's in a field outside of STEM, she could be more of an unbiased perspective as to why women and minorities have a low retention rate in STEM.

Relevant quotes:
“However, it is still not well understood exactly what factors affect persistence in STEM majors during college, and therefore where the focus should be placed in order to improve persistence.”

“The fraction of female and/or minority faculty in STEM departments is very low, so if role models of the same gender or race are important factors in major choice, this could play an important role in the under-representation of women and minorities in STEM field majors.”

“A higher percentage of faculty members that are female would therefore measure more opportunity for female students to identify with female role models in the field, or to connect with a female mentor in the field.”

“48% of the students in the NLSF sample that stay in a STEM major until at least their sophomore year of college are female, while 54% of students in the NLSF that switch to a non-STEM major are female.”

“Planned STEM majors, as well as those that persist to a STEM major, tend to have higher high school GPAs on average, and to have taken a larger percentage of their AP courses in STEM fields, than do students that planned to major in a non-STEM field or did not persist.”

5.4 McNeill 2019

Citation: McNeill, R. Taylor. "19 “I Wish I Could Say ‘You Should Not be Here’”: An Analysis of Instructors’ and Students’ Contrasting Perceptions of a Racialized and Gendered Gatekeeping Practice in Calculus." *Justice Through the Lens of Calculus*: 287.

Summary: This is more of a documentation of gatekeeping in the field where a professor has a class-wide message telling the students that they do not belong in the classroom if they can not complete a problem with a certain speed. This is more of a discussion on intent because the professor means it as a motivating speech, but some students feel that it makes the field exclusionary. The paper’s main goal is to use these events to bring to light the need for professors to be more self-aware about how they talk about mathematics and their students. The study is more survey style where they ask students to recount certain events that made them feel a certain way about the field and separated them into different categories such as colorblind or genderblindness, problematizing student motivation or belonging in STEM, etc.

Relevant quotes:
“Parker describes how racial stereotyping of mathematical ability would make him particularly vulnerable to feeling discouraged if he experienced the class-wide course drop event. Although Parker continued with mathematics, prior research has shown that racialized and gendered stereotypes of mathematics ability can contribute to student attrition from STEM (Beasley & Fischer, 2012).”

“...situations like the course drop event can perpetuate exclusionary ideas of who belongs and succeeds in mathematical spaces,”

"...racial consciousness is necessary to disrupt norms and practices in STEM which perpetuate racial oppression (Haynes & Patton, 2019). However, relying on individual faculty to develop critical consciousness and reform their own teaching practice may be inadequate since it has been found that mathematics faculty, as a group, are poor advocates for diversity (Park & Denson, 2009) and often lack professional training and incentives to cultivate equitable teaching practices (Haynes & Patton, 2019; Walczyk et al., 2005)."

5.5 Steele et al 2005

Citation: Steele, D.F., Levin, A.K., Blecksmith, R. *et al.* The calculus of differences: Effects of a psychosocial, cultural, and pedagogical intervention in an all women's university calculus class. *Math Ed Res J* **17**, 22–44 (2005). https://doi.org/10.1007/BF03217420

Summary: This paper talks about how they used connect learning to incorporate mathematics concepts with things that the students will be interacting with day to day to understand calculus fully. They explain that some women steer away from math-related courses because of the competitive nature and the feeling of inadequacy in the classroom. A group called FIG (Focused Interest Group) was created using connect learning to help women grow confident in their math skills; the researchers found out that a lot more women were staying in math-related majors when they were in the FIG group than not because they had a support system in the professors and other women in the group. There are four authors, and two of them were directly in the study as a professor (Richard Blecksmith) and as an interviewer (Jill Shahverdian). There are two women who were emphasized as both having positive experiences joining the FIG group with different backgrounds and slightly different experiences in the group. Blecksmith was the trusted male professor for the FIG group and seemed to have a big impact on the support of the students, and Jill Shahverdian was present to listen to and support the students outside of the classroom. She is also a professor for the Mathematics and Women's and Gender Studies Department. Diana F. Steele has written papers about women in mathematics and using sociocultural theory to teach mathematics. Amy K. Levin is the chair of the English department and directs the women's study program.

Relevant Quotes:

"...connected knowers learn through making connections. These learners want to form a relationship between themselves, their peers, and ideas. Becker (1996) said that "[c]onnected knowers focus on the context and other people's knowledge" (p. 20). The connected knower finds it helpful to maintain a group connection where she can learn to create her own system of knowing and develop her own authorities." (*p. 26)*

"The curriculum humanised mathematics; mathematical concepts were placed in context through problems that connected to students' interests, experiences, and relationships." (*p. 28)*

"...the professor who taught the intervention class believed that he had taught the students in more depth and expected more rigor from them in their regular hourly examinations than in his past Calculus I classes. He claimed that he taught the group as if they were in the honours section of calculus. The students also performed comparably to students in other sections on the common department final examination. The final examination did not include the conceptual types of problems in context that the

professor predominantly had used in their calculus class and hourly examinations, yet the students' learning carried over." (*p. 32)*

5.6 Yosso 2006

Citation: Yosso *, T. J. (2005). Whose culture has capital? A critical race theory discussion of community cultural wealth. *Race Ethnicity and Education*, *8*(1), 69–91. https://doi.org/10.1080/1361332052000341006

Summary: This journal talks about critical race theory and cultural wealth, which explains that the knowledge that students of color bring from their communities and outside experiences helps them in school. The authors discuss critiquing deficit theorizing that may limit the voices of BIPOC people. They explain that students have their cultural wealth to help them understand concepts. Yosso is an assistant dean of academic personnel, applying the framework of critical race theory in education.

Relevant quotes:

"Pierre Bourdieu argued that the knowledges of the upper and middle classes are considered capital valuable to a hierarchical society."

"Bourdieu's theoretical insight about how a hierarchical society reproduces itself has often been interpreted as a way to explain why the academic and social outcomes of People of Color are significantly lower than the outcomes of Whites."

"As part of the challenge to deficit thinking in education, it should be noted that race is often coded as 'cultural difference' in schools. Indeed, culture influences how society is organized, how school curriculum is developed and how pedagogy and policy are implemented."