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STUDENT VERSION

Projectile Motions

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Abstract:

We consider several instances of projectile flight without resistance, one on level ground and one from edge of cliff to determine maximum distance and placement. Namely, determine the maximum range of the projectile in each case and in the first case what launch angle should be used to hit a target which is within range.

SCENARIO DESCRIPTION

Way up! Far out! Spot on! Way down!

Consider a projectile of mass m kg, say a small steel ball, launched from the point $P = (0, 0)$ m on an x-y-axis with an initial velocity of v_0 m/s at a launch angle θ off the x-axis and in the direction of increasing x values as depicted in Figure 1.

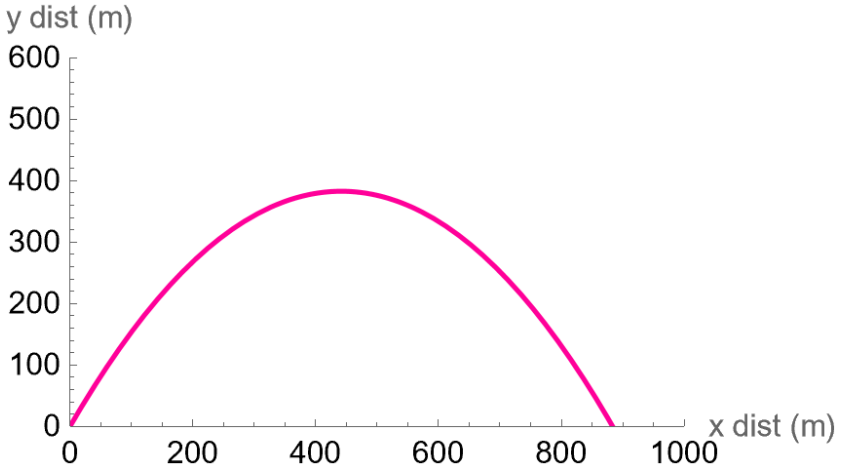

Figure 1. Sample projectile plot.

Offer your analysis in each situation, all of which consider flight without resistance due to air.

(a) For given initial velocity, $v_0 = 100$ m/s, what launch angle θ will send the projectile farthest downrange horizontally?

(b) Supposing we know our initial muzzle velocity, $v_0 = 100$ m/s. At what angle θ will we have to launch so as to hit a given spot down range? Indeed, how will we know if we can even reach the given spot?

(c) Suppose the launcher is still at point $P = (0, 0)$ m, but that point P is on the edge of a precipitous cliff of height $hc = 100$ m. Further, suppose $v_0 = 100$ m/s and $g = 9.81m/s^2$. Now what launch angle θ will send the projectile farthest horizontally along the flat plain below the cliff?

Further questions might be in part (c) to determine the exact launch angle to strike a downrange point on the x-axis Also, consider what happens when we are in more realistic situation of resistance or the projectile is launched in a stiff wind.