Real-World Context

Students divide into groups, and each group imagines rebuilding the school playground. Students may brainstorm one of many different modeling problems:

- What kind of playground would they want at school?
- How many of each piece of equipment should there be?
- What is the most efficient way to organize the playground? Is efficiency a measure of area, time to travel between parts of the playground, minimized cost, or something else (maybe a combination of several factors)?
- How can you minimize cost of the new playground without reducing the number of kids who can play on it at the same time?

Possible Math Tools

3 – 5 students: Use of the four operations, calculations of area and perimeter, data collection, working with money, estimation, rates, unit conversions

Relevant Common Core Standards:

**CCSS.MATH.CONTENT.4.OA.A**
Use the four operations with whole numbers to solve problems.

**Task:** Calculate how many students could use your playground at one time.

**CCSS.MATH.CONTENT.4.NBT.B**
Use place value understanding and properties of operations to perform multi-digit arithmetic.

**Task:** Determine what the cost should be for the playground and how the cost changes for different equipment choices, given costs of pieces of equipment and the number of each to be bought.

**CCSS.MATH.CONTENT.4.MD.A**
Solve problems involving measurement and conversion of measurements.

**Task:** Convert all units of dimension given for the various playground equipment pieces to the same unit system (ex. feet to meters).

**CCSS.MATH.CONTENT.4.MD.B**
Represent and interpret data.

**Task:** Design a survey for classmates about which equipment to purchase and display the results in some form of graph or table.

**CCSS.MATH.CONTENT.4.G.A**
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

**Task:** Identify shapes and angles within the different pieces of playground equipment in use.

**CCSS.MATH.CONTENT.5.NBT.B**
Perform operations with multi-digit whole numbers and with decimals to hundredths.
Task: Include a fence around the playground in the design and incorporate the cost of the fence into the budget given a cost of the fence per unit length.

CCSS.MATH.CONTENT.5.NF.B
Apply and extend previous understandings of multiplication and division.

Task: Calculate the area of the playground the chosen playground items will cover.

CCSS.MATH.CONTENT.5.MD.A
Convert like measurement units within a given measurement system.

Task: If some of the playground items dimensions are given in yards or inches, convert them to feet.

CCSS.MATH.CONTENT.5.MD.B
Represent and interpret data.

Task: Draw the playground layout using the first quadrant of the coordinate plane and name the positions of the corners of the shapes used to represent the equipment using with ordered pairs.
**Dive In:**
Students begin exploring the topic.

<table>
<thead>
<tr>
<th>Student Actions</th>
<th>Teacher Actions</th>
</tr>
</thead>
</table>
| Students will explore the topic by answering questions such as:  
  - What do you notice? What do you wonder?  
  - What is interesting about this topic?  
  - What about this topic is important?  
  - What information do you need?  
| What will you show/tell students to launch the real-world context and capture their interest?  
  - Show students pictures of different playgrounds, discuss aspects of playgrounds they’ve liked and disliked in the past.  
    - Get them to think about how they might redesign the playground.  
  - During the launch, similar lessons have brought in notable school figures such as the principal to speak to the class and motivate their endeavors.  
  - Direct the students towards not only thinking about what kinds of playgrounds they like, but how they can more formally evaluate what makes a good playground.  
    - Is more variety better?  
    - Are highly desirable playground items with long wait times better or slightly less desirable items with shorter waiting times?  
  - If you chose to pose the problem with a fixed budget, consider introducing that budget somewhere in the end of the launch.  
| Students will brainstorm these questions in groups.  
| Allow students time to brainstorm. Monitor student progress and group dynamics.  
| Take note of anything that should be shared with the class:  
  - ideas that help students mathematize the problem  
  - common misconceptions |
Define the Problem:
Ideas are narrowed to a focused, mathematically relevant problem.

<table>
<thead>
<tr>
<th>Student Actions</th>
<th>Teacher Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will choose a focused problem that can be answered and justified with information and mathematics.</td>
<td>Guide students towards a focused problem that can be answered and justified with information and mathematics.</td>
</tr>
<tr>
<td>Students should consider questions such as:</td>
<td></td>
</tr>
<tr>
<td>• What information do you need to make a model?</td>
<td><em>What are my expectations for the model? Will the whole class focus on the same problem, or will variation be allowed?</em></td>
</tr>
<tr>
<td>• What quantities are required by the model? Which ones are provided?</td>
<td>*<em>As a class, decide which types of playground items should be allowed in the design. Let groups make their own models, but constrain everyone to these playground options. However, students may decide on different versions of “efficiency”.</em></td>
</tr>
<tr>
<td>• Do quantities have only one value, or can they have a range of values?</td>
<td></td>
</tr>
<tr>
<td>• What mathematical tools could you use in your model?</td>
<td><em>What mathematical tools/connections could you suggest to students who aren’t using math?</em></td>
</tr>
<tr>
<td></td>
<td>• Have students think about rates (consider how long it takes to use an item in the playground, how many students can use an item at once, or even the whole playground at once.)</td>
</tr>
<tr>
<td></td>
<td>• Encourage thinking about comparisons (how much students like a given item?)</td>
</tr>
<tr>
<td></td>
<td>• Consider adding up areas and costs. (Will these items fit in the playground? Do we have enough money to buy them?)</td>
</tr>
<tr>
<td></td>
<td>• Narrow the problem by giving students a specific set of playground items with dimensions and prices to choose from. By limiting students’ focus to a few items, they’ll focus less on the design component and more on the math.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>How will you guide your students to use new skills they are less comfortable with?</em></td>
</tr>
<tr>
<td></td>
<td>• If you want to emphasize measurement and blueprint drawing skills, you may want to have the students go outside and measure and draw the aerial layout of the playground themselves. Otherwise, consider giving students a layout of the playground space they have to work with.</td>
</tr>
<tr>
<td></td>
<td>• Students may also consider measuring playground item popularity by counting the number of times each item is used over a set period or by collecting their classmates’ preferences through surveys.</td>
</tr>
</tbody>
</table>
Do the Math:
Iterate the model until it is done and can be evaluated.

<table>
<thead>
<tr>
<th>Student Actions</th>
<th>Teacher Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use mathematical tools to develop a model.</td>
<td>Note the mathematics that develops during model building.</td>
</tr>
<tr>
<td>Mathematically justify all estimations and numerical values in model.</td>
<td><strong>What are some common misconceptions that could arise at this stage, and how might you address them?</strong></td>
</tr>
<tr>
<td>Use the model to suggest a solution.</td>
<td></td>
</tr>
<tr>
<td>Record work.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| | • **Students may not understand the difference between an estimate and an exact value**  
| |   o Ask if 21 kids used the slide today, will exactly 21 use the slide every day  
| | • **Students may struggle to use their information to calculate rates**  
| |   o Ask how many people can use this item in one recess period and how long recess is  
| | • **Students may be unsure of how to measure areas of non-rectangular playground regions**  
| |   o Ask if they can combine non-square shapes into full squares (e.g. two triangles)  
| | Address misconceptions individually or as a group.  
| | **When are natural times to regroup?**  
| | After data collection, if groups are struggling with rate calculations or other model factors
**Decide Whether You’re Satisfied, and Declare Victory:**
Evaluate your model and decide when the model is ready to be presented.

<table>
<thead>
<tr>
<th>Student Actions</th>
<th>Teacher Actions</th>
</tr>
</thead>
</table>
| Students should be evaluating their model by asking questions such as:  
- If there is a rubric or checklist, see if you did everything.  
- Is your solution reasonable? Why or why not?  
- Is your solution useful for answering your question?  | **What components do you expect the students’ models to include?**  
The model should include a blueprint and budget. The budget should determine which items to purchase based on a ranking system of some kind. The model should also predict how many people can enjoy the playground at once. That question could be answered by considering how many people can use their favorite piece of playground equipment at once.  |

**What will a useful model be able to do?**
- A useful model should do at least one of the following:  
  - Try to let as many students play on the playground as possible  
  - Try to put as many of students’ favorite items in the playground as possible  
  - Try to fit as much in the playground space as possible while keeping in budget  
  - Some other clearly defined measure of efficiency  
- Overall, the model should try to help students decide on what playground items should be bought based off of information they can collect (student enjoyment, rate at which students can use a playground item, amount of space an item takes up etc.)  

**Define an ending point** for your students’ models, and set clear expectations.

Guide students through reviewing their models by considering the questions on the left.
**Demonstrate Solution:**
Present and interpret your model that solves the problem.

<table>
<thead>
<tr>
<th>Student Actions</th>
<th>Teacher Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will reflect, justify, and present their models by asking and answering questions such as:</td>
<td><strong>What expectations do you have for students’ presentations?</strong></td>
</tr>
<tr>
<td>- Why would you recommend your model to someone?</td>
<td>- A good model should discuss how students decided to evaluate what makes a good playground.</td>
</tr>
<tr>
<td>- What mathematical tools did you use, and how did they help solve the problem?</td>
<td>- This includes how they took measurements to gather information on the playground and how they used that information to make their model and decide which playground items to include in their final design.</td>
</tr>
<tr>
<td>- What did you change in your model throughout the modeling process?</td>
<td>- The solutions should describe their final solution using the playground layout they created.</td>
</tr>
<tr>
<td>- Are there situations where your solution wouldn’t work or your model wouldn’t apply?</td>
<td>Guide students in evaluating their solutions by answering the questions on the left, as a whole class or in groups.</td>
</tr>
<tr>
<td>- How would you need to change your model to apply to more situations?</td>
<td>(A presentation rubric from IMMERSION is available on the Math Modeling Hub.)</td>
</tr>
<tr>
<td>- If you had more time, what else would you do?</td>
<td></td>
</tr>
<tr>
<td>- Are there any mathematical tools or pieces of information that would have been helpful to have?</td>
<td></td>
</tr>
</tbody>
</table>
Revisit:
These questions may help you consider possible extensions to the problem. Tying the problem to more advanced math gives students a frame of reference for newer mathematical tools.

Q: When could you recall the math used in this lesson as a starting point or an example later in your curriculum?
A: This problem could apply to any model where rates and multiple factors that can be considered for efficiency must both be considered. More complicated modeling problems may refer to this one as an example of the necessary methods.

Q: Is there a time later in the year when you might come back to this real-world scenario with different mathematical tools? Remember that students sometimes reach for tools that are most familiar and it might take them a while to build confidence to use a new tool in a modeling situation.
A: This problem already uses most of the skills students will learn at this time. A revisit with more math skills is not necessarily required.

Q: Throughout the year, will you be collecting new information about this scenario? Are there times you could use that information to reflect on and improve your model?
A: Information should be collected over the course of one day. However, if the school designs a new playground, or buys new equipment (especially if the students’ suggestions were taken into account) that may be a time to return to this problem considering the new playground equipment.

Q: Are there other similar scenarios where you could use the same kinds of models? What might change? What might stay the same?
A: Any project where efficiency is being considered can use this kind of modeling. Different projects will change which metrics are being assessed for efficiency and how to compare them, but the process will remain the same.

For more resources on how to change parameters and constraints or how to extend this task to other grades, consider consulting the GAIMME report pages 136-139 http://www.siam.org/reports/gaimme.php.