Design a Zoo
Lesson Plan

Real-World Context	Possible Math Tools
Students divide into groups, and each group designs a zoo: it must have more than one exhibit, and students must consider multiple sources of cost. Students may brainstorm one of many different modeling problems: • How much should we charge for admittance? • How could we reduce an outstanding cost? • How can we design a profitable zoo with limited space? • If we went on a field trip to the zoo, could we learn which exhibits are popular? Which one is the best investment?	6 – 8 students: Data analysis, linear functions, data extrapolation

Relevant Common Core Standards:

CCSS.MATH.CONTENT.6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
Task: Construct a formula for the cost of a zoo, starting with cost per animal, then cost per exhibit.

CCSS.MATH.CONTENT.6.EE.A.2
Write, read, and evaluate expressions in which letters stand for numbers.
Task: Find the cost and profit of a zoo in which some quantities are unknown. Test different quantities and find the best option.

CCSS.MATH.CONTENT.6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
Task: Develop a function of cost and revenue against time.

CCSS.MATH.CONTENT.6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
Task: Collect data on the popularity of different exhibits, and use the data to plan which exhibits should get the most focus.

CCSS.MATH.CONTENT.7.RP.A.2
Recognize and represent proportional relationships between quantities.
Task: Construct a generalized cost formula, and test it with specific values.

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CCSS.MATH.CONTENT.8.EE.B.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
Task: Graph the cost of a zoo over time. Compare the results of different groups, to find when the cost of one overtakes another.

CCSS.MATH.CONTENT.8.EE.C.8
Analyze and solve pairs of simultaneous linear equations.
Task: Graph the cost of a zoo over time. Compare the results of different groups, to find when the cost of one overtakes another.

Dive In:

Students begin exploring the topic.

Student Actions	Teacher Actions
Students will explore the topic by answering questions such as: • What do you notice? What do you wonder? • What is interesting about this topic? • What about this topic is important? • What information do you need? Students will brainstorm these questions in groups.	*What will you show/tell students to launch the real-world context and capture their interest?* • **Discuss a visit to the zoo. What can be seen there? How many exhibits do students remember? What makes them different?** • **Next, ask if students have any pets. What are the costs associated with owning a pet? Food? Cleanup? What about time commitment?** Allow students time to brainstorm. Monitor student progress and group dynamics. Take note of anything that should be shared with the class: • ideas that help students mathematize the problem • common misconceptions

Define the Problem:

Ideas are narrowed to a focused, mathematically relevant problem.

Student Actions	Teacher Actions
Students will choose a focused problem that can be answered and justified with information and mathematics. Students should consider questions such as: • What information do you need to make a model? • What quantities are required by the model? Which ones are provided? • Do quantities have only one value, or can they have a range of values? • What mathematical tools could you use in your model?	Guide students towards a focused problem that can be answered and justified with information and mathematics. *What are my expectations for the model? Will the whole class focus on the same problem, or will variation be allowed?* **Different groups may have different ways of calculating cost. For example, one group may focus on building and maintaining the habitats, while another group may focus on providing food and other care.** *What mathematical tools/connections could you suggest to students who aren't using math?* **Ask students to think of two exhibits, one with a higher initial cost, one with a higher daily cost. Encourage them to use multiplication or linear functions to find which one is most cost-effective.** *How will you guide your students to use new skills they are less comfortable with?* **Encourage students to consider the cost of exhibits after some time (e.g. one week) using multiplication.**

Do the Math:

Iterate the model until it is done and can be evaluated.

Student Actions	Teacher Actions
Use mathematical tools to develop a model. Mathematically justify all estimations and numerical values in model. Use the model to suggest a solution. Record work.	Note the mathematics that develops during model building. *What are some common misconceptions that could arise at this stage, and how might you address them?* **Some costs are only paid once, while others are daily, weekly, or monthly; students may not remember this. Ask students to consider what their zoo would be like in a year: what would they still be paying?** Address misconceptions individually or as a group. *When are natural times to regroup?* **After the students have completed drafts: they should know what exhibits they want to include, estimate their costs, and conduct surveys on which ones are popular.**

Decide Whether You're Satisfied, and Declare Victory:

Evaluate your model and decide when the model is ready to be presented.

Student Actions	Teacher Actions
Students should be evaluating their model by asking questions such as: • If there is a rubric or checklist, see if you did everything. • Is your solution reasonable? Why or why not? • Is your solution useful for answering your question?	*What components do you expect the students' models to include?* **A map of the zoo, a table of costs, and a suggested price of admission.** *What will a useful model be able to do?* **It must present a way to reallocate space in the zoo for improved profit, and show the quantitative difference.** **Define an ending point** for your students' models, and set clear expectations. Guide students through reviewing their models by considering the questions on the left.

Demonstrate Solution:

Present and interpret your model that solves the problem.

Student Actions	Teacher Actions
Students will reflect, justify, and present their models by asking and answering questions such as: • Why would you recommend your model to someone? • What mathematical tools did you use, and how did they help solve the problem? • What did you change in your model throughout the modeling process? • Are there situations where your solution wouldn't work or your model wouldn't apply? • How would you need to change your model to apply to more situations? • If you had more time, what else would you do? • Are there any mathematical tools or pieces of information that would have been helpful to have?	*What expectations do you have for students' presentations?* **Groups should be able to properly explain the decisions they made, and how they relate to the mathematical topics of interest.** Guide students in evaluating their solutions by answering the questions on the left, as a whole class or in groups. (A presentation rubric from IMMERSION is available on the Math Modeling Hub.)

Revisit:

These questions may help you consider possible extensions to the problem. Tying the problem to more advanced math gives students a frame of reference for newer mathematical tools.

Q: *When could you recall the math used in this lesson as a starting point or an example later in your curriculum?*

A: **This is problem could apply to any model where "initial costs" and "daily costs" must both be considered. More complicated modeling problems may refer to this one as an example of the necessary methods.**

Q: *Is there a time later in the year when you might come back to this real-world scenario with different mathematical tools? Remember that students sometimes reach for tools that are most familiar and it might take them a while to build confidence to use a new tool in a modeling situation.*

A: **As students learn more linear algebra, they can discover new techniques for predicting and optimizing cost.**

Q: *Throughout the year, will you be collecting new information about this scenario? Are there times you could use that information to reflect on and improve your model?*

A: **Information should be collected in a single day.**

Q: *Are there other similar scenarios where you could use the same kinds of models? What might change? What might stay the same?*

A: **Replace "zoo" with "shopping center," "museum," or one of many other businesses. The same methods may be used, but the sources of cost should work differently, perhaps even nonlinearly.**

For more resources on how to change parameters and constraints or how to extend this task to other grades, consider consulting the GAIMME report pages 136-139 http://www.siam.org/reports/gaimme.php.