Beanbag Toss
Lesson Plan

Real-World Context	Possible Math Tools
Students divide into groups, and each group designs a beanbag toss game. The game must be fair enough to attract players, and challenging enough to keep them invested. Students use the resources at their disposal to design a carnival game, and use data to set an appropriate level of challenge by changing player accuracy. Students may brainstorm one of many different modeling problems: • How big should the target be? • How far should a player stand from the target? • What kind of obstacles should be in the way?	6 – 8 students: Functions, probabilities and distributions

Relevant Common Core Standards:

CCSS.MATH.CONTENT.6.SP.A.1
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
Task: Prepare statistical questions to determine the accuracy of the average player.

CCSS.MATH.CONTENT.6.SP.A.3
Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Task: Describe the accuracy of a set of parameters for the beanbag toss (i.e. with a target of size X and a person Y meters away) as a statistical grouping (the mean accuracy is Z with a standard deviation of W).

CCSS.MATH.CONTENT.6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
Task: Compare data visually and determine which parameters make the game both fair and challenging.

CCSS.MATH.CONTENT.6.SP.B.5
Summarize numerical data sets in relation to their context.
Task: Interpret what the collected data means in terms of the beanbag toss game.

CCSS.MATH.CONTENT.7.SP.C.5
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
Task: Calculate the probability of hitting the target in each version of the beanbag toss game.

Dive In:

Students begin exploring the topic.

Student Actions	Teacher Actions
Students will explore the topic by answering questions such as: • What do you notice? What do you wonder? • What is interesting about this topic? • What about this topic is important? • What information do you need? Students will brainstorm these questions in groups.	*What will you show/tell students to launch the real-world context and capture their interest?* **Allow students to talk about their experience with games, and ask what makes them fun. How can games be quantified? What can be added or changed? Consider the beanbag toss specifically.** Allow students time to brainstorm. Monitor student progress and group dynamics. Take note of anything that should be shared with the class: • ideas that help students mathematize the problem • common misconceptions

Define the Problem:

Ideas are narrowed to a focused, mathematically relevant problem.

Student Actions	Teacher Actions
Students will choose a focused problem that can be answered and justified with information and mathematics. Students should consider questions such as: • What information do you need to make a model? • What quantities are required by the model? Which ones are provided? • Do quantities have only one value, or can they have a range of values? • What mathematical tools could you use in your model?	Guide students towards a focused problem that can be answered and justified with information and mathematics. *What are my expectations for the model? Will the whole class focus on the same problem, or will variation be allowed?* **Different groups should examine different qualities or rules of the game. The students can choose whichever one they want, as long as it can be measured and varied.** *What mathematical tools/connections could you suggest to students who aren't using math?* **Ask students to plot their results on a line or scatter graph. Does the graph seem to have a certain shape? What does the graph indicate? What function does it resemble most closely?** *How will you guide your students to use new skills they are less comfortable with?* **If the students aren't sure how to interpret the data, encourage them to fill out the provided table, and try to describe the resulting pattern. Graphing may help with this.**

Do the Math:
Iterate the model until it is done and can be evaluated.

Student Actions	Teacher Actions
Use mathematical tools to develop a model. Mathematically justify all estimations and numerical values in model. Use the model to suggest a solution. Record work.	Note the mathematics that develops during model building. *What are some common misconceptions that could arise at this stage, and how might you address them?* **Students may not understand the purpose of taking data at several different settings. Point out the difference between an estimate and an exact value.** Address misconceptions individually or as a group. *When are natural times to regroup?* **Once the students have some experience running tests, or when they're ready to interpret their data.**

Decide Whether You're Satisfied, and Declare Victory:
Evaluate your model and decide when the model is ready to be presented.

Student Actions	Teacher Actions
Students should be evaluating their model by asking questions such as: • If there is a rubric or checklist, see if you did everything. • Is your solution reasonable? Why or why not? • Is your solution useful for answering your question?	*What components do you expect the students' models to include?* **A succinct description of how they designed their game, and how challenging it is.** *What will a useful model be able to do?* **Define some quality of the game, and show how it correlates with a player's rate of success.** **Define an ending point** for your students' models, and set clear expectations. Guide students through reviewing their models by considering the questions on the left.

Demonstrate Solution:

Present and interpret your model that solves the problem.

Student Actions	Teacher Actions
Students will reflect, justify, and present their models by asking and answering questions such as: • Why would you recommend your model to someone? • What mathematical tools did you use, and how did they help solve the problem? • What did you change in your model throughout the modeling process? • Are there situations where your solution wouldn't work or your model wouldn't apply? • How would you need to change your model to apply to more situations? • If you had more time, what else would you do? • Are there any mathematical tools or pieces of information that would have been helpful to have?	*What expectations do you have for students' presentations?* **Groups should be able to reasonably explain how they chose their trials, and demonstrate how they can adapt their game to various levels of challenge.** Guide students in evaluating their solutions by answering the questions on the left, as a whole class or in groups. (A presentation rubric from IMMERSION is available on the Math Modeling Hub.)

Revisit:

These questions may help you consider possible extensions to the problem. Tying the problem to more advanced math gives students a frame of reference for newer mathematical tools.

Q: *When could you recall the math used in this lesson as a starting point or an example later in your curriculum?*

A: **The method of taking data can be recalled in future statistics lessons.**

Q: *Is there a time later in the year when you might come back to this real-world scenario with different mathematical tools? Remember that students sometimes reach for tools that are most familiar and it might take them a while to build confidence to use a new tool in a modeling situation.*

A: **Students who are well-versed in statistics may want to revisit this problem with better data, and even try to find linear relationships between certain qualities.**

Q: *Throughout the year, will you be collecting new information about this scenario? Are there times you could use that information to reflect on and improve your model?*

A: **No, but students are encouraged to build bigger and more complicated games at home.**

Q: *Are there other similar scenarios where you could use the same kinds of models? What might change? What might stay the same?*

A: **This is a basic application of probability distributions, which may be similar to a future 6 – 8 curriculum.**

For more resources on how to change parameters and constraints or how to extend this task to other grades, consider consulting the GAIMME report pages 136-139 http://www.siam.org/reports/gaimme.php.