Interspecific Competition: the Lotka-Volterra Model

# Introduction

Interspecific competition refers to the competition between two or more species for some limiting resource. This limiting resource can be food or nutrients, space, mates, nesting sites-- anything for which demand is greater than supply. When one species is a better competitor, interspecific competition negatively influences the other species by reducing population sizes and/or growth rates, which in turn affects the population dynamics of the competitor. The Lotka-Volterra model of interspecific competition is a simple mathematical model that can be used to understand how different factors affect the outcomes of competitive interactions.

# Importance

Competitive interactions between organisms can have a great deal of influence on species evolution, the structuring of communities (which species coexist, which don't, relative abundances, etc.), and the distributions of species (where they occur). Modeling these interactions provides a useful framework for predicting outcomes.

# Questions

Under what circumstances can two species coexist? Under what circumstances does one species outcompete another?

# Variables

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| N | population size |
| t | time |
| K | carrying capacity |
| r | intrinsic rate of increase |
|  | competition coefficient |

# Methods

The logistic equation below models a rate of population increase that is limited by intraspecific competition (i.e., members of the same species competing with one another).

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| $$\frac{dN}{dt}=rN\frac{\left(K-N\right)}{K}$$ | LaTeX Code: \[ \frac{dN}{dt} = rN \frac{(K-N)}{K} \] |

The first term on the right side of the equation (*rN*, the intrinsic rate of increase [*r*] times the population size [*N*]) describes a population's growth in the absence of competition. The second term ([*K*-*N*] / *K*) incorporates intraspecific competition, or density-dependence, into the model, and takes a value between 0 and 1. As population size (*N*) approaches carrying capacity (*K*), the numerator (*K*-*N*) becomes smaller but the denominator (*K*) stays the same and the second term decreases. The addition of this term describes a rate of population growth that slows down as population size increases, until the population reaches its carrying capacity. In other words, the growth curve described by the logistic equation is sigmoidal, and the rate of growth depends on the density of the population.

The logistic equation can be modified to include the effects of interspecific competition as well as intraspecific competition. The Lotka-Volterra model of interspecific competition is comprised of the following equations for population 1 and population 2, respectively:

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| $$\frac{dN\_{1}}{dt}=r\_{1}N\_{1}\frac{\left(K\_{1}-N\_{1}-α\_{12}N\_{2}\right)}{K\_{1}}$$ | LaTeX Code: \[ \frac{dN\_1}{dt} = r\_1 N\_1 \frac{(K\_1 - N\_1 - \alpha\_{12}N\_2)}{K\_1} \] |
| $$\frac{dN\_{2}}{dt}=r\_{2}N\_{2}\frac{\left(K\_{2}-N\_{2}-α\_{21}N\_{1}\right)}{K\_{2}}$$ | \[ \frac{dN\_2}{dt} = r\_2 N\_2 \frac{(K\_2 - N\_2 - \alpha\_{21}N\_1)}{K\_2} \] |

The big difference (other than the subscripts denoting populations 1 and 2) is the addition of a term involving the competition coefficient, . The competition coefficient represents the effect that one species has on the other: 12 represents the effect of species 2 on species 1, and 21 represents the effect of species 1 on species 2 (the first number of the subscript always refers to the species being affected). In the first equation of the Lotka-Volterra model of interspecific competition, the effect that species 2 has on species 1 (12) is multiplied by the population size of species 2 (*N*2). When 12 is < 1 the effect of species 2 on species 1 is less than the effect of species 1 on its own members. Conversely, when 12 is > 1 the effect of species 2 on species 1 is greater than the effect of species 1 on its own members. The product of the competition coefficient, 12, and the population size of species 2, *N*2, therefore represents the effect of an equivalent number of individuals of species 1, and is included in the intraspecific competition, or density-dependence, term. The 21*N*1 term in the second equation is interpreted in the same way.

To understand the predictions of the model it is helpful to look at graphs that show how the size of each population increases or decreases when we start with different combinations of species abundances (i.e., low *N*1 low *N*2, high *N*1 low *N*2, etc.). These graphs are called state-space graphs, in which the abundance of species 1 is plotted on the *x*-axis and the abundance of species 2 is plotted on the *y*-axis. Each point in a state-space graph represents a combination of abundances of the two species. For each species there is a straight line on the graph called a zero isocline. Any given point along, for example, species 1's zero isocline represents a combination of abundances of the two species where the species 1 population does not increase or decrease (the zero isocline for a species is calculated by setting *dN/dt*, the growth rate, equal to zero and solving for *N*). The two graphs below show the zero isoclines for species 1 (left, solid yellow line) and species 2 (right, dashed pink line). (All graphs adapted from Begon et al. [1996] and Gotelli [1998])

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| http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph1.gif | http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph2.gif |

Note that the zero isoclines divide each graph into two parts. Below and to the left of the isocline the population size increases because the combined abundances of both species are less than the carrying capacity of the one, while above and to the right the population size decreases because the combined abundances are greater than the carrying capacity. For the graph of the isocline of species 1, the isocline intersects the graph on the *x*-axis when *N*1 reaches its carrying capacity (*K*1) and no individuals of species 2 are present. The isocline intersects the graph on the *y*-axis at *K*1/12, when the carrying capacity of species 1 is filled by the equivalent number of individuals of species 2 and no individuals of species 1 are present. The intersections of the isocline for species 2 are essentially the same, but on different axes.

These two graphs illustrate what happens to a population when it is below or above its isocline, but they only account for one isocline at a time. The following four graphs include both species' isoclines and illustrate the possible outcomes of interspecific competition depending on where each species' isocline lies in relation to the other. In each graph, the solid yellow line represents the isocline of species 1, and the dashed pink line represents the isocline of species 2. The thick black arrows represent the joint trajectory of the two populations, and the thinner colored arrows indicate the trajectories of the individual populations.

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| http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph3.gif | http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph4.gif |

# Interpretation

The first scenario is one in which the isocline for species 1 is above and to the right of the isocline for species two. For any point in the lower left corner of the graph (i.e., any combination of species abundances), both populations are below their respective isoclines and both increase. For any point in the upper right corner of the graph, both species are above their respective isoclines and both decrease. For any point in between the two isoclines, species 1 is still below its isocline and increases, while species 2 is above its isocline and decreases. The joint movement of the two populations (thick black arrows) is down and to the right, so species 2 is driven to extinction and species 1 increases until it reaches carrying capacity (*K*1). The open circle at this point represents a stable equilibrium. In this scenario, species 1 always outcompetes species 2, and is referred to as the competitive exclusion of species 2 by species 1.

The second scenario is the opposite of the first; the isocline of species 2 is above and to the right of the isocline for species 1. This graph can be interpreted in much the same way as the previous one, except that the joint trajectory of the two populations when starting in between the isoclines is up and to the left. In this case species 2 always outcompetes species 1, and species 1 is competitively excluded by species 2.

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| http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph5.gif | http://www.tiem.utk.edu/~gross/bioed/bealsmodules/competition.gph6.gif |

In the third scenario, the isoclines of the two species cross one another. Here, the carrying capacity of species 1 (*K*1) is higher than the carrying capacity of species 2 divided by the competition coefficient (*K*2/21), and the carrying capacity of species 2 (*K*2) is higher than the carrying capacity of species 1 divided by the competition coefficient (*K*1/12). Below both isoclines and above both isoclines the populations increase or decrease as in the first two scenarios, and there is an unstable equilibrium point (closed circle) where the isoclines intersect. For points above the dashed pink line (species 2 isocline) and below the solid yellow line (species 1 isocline), the outcome is the same as in the first scenario: competitive exclusion of species 2 by species 1. On the other hand, for points above the solid yellow line (species 1 isocline) and below the dashed pink line (species 2 isocline), the outcome is the same as in the second scenario: competitive exclusion of species 1 by species 2. The two stable equilibrium points are again represented by open circles. In this scenario, the outcome depends on the initial abundances of the two species.

Finally, in the fourth scenario we can see that the isoclines cross one another, but in this case both species' carrying capacities are lower than the other's carrying capacity divided by the competition coefficient. Again, below both isoclines the populations increase and above both isoclines the populations decrease. In this case, however, when the populations of the two species are between the isoclines their joint trajectories always head toward the intersection of the isoclines. Rather than outcompeting one another, the two species are able to coexist at this stable equilibrium point (open circle). This is the outcome regardless of the initial abundances.

# Conclusion

The Lotka-Volterra model of interspecific competition has been a useful starting point for biologists thinking about the outcomes of competitive interactions between species. The assumptions of the model (e.g., there can be no migration and the carrying capacities and competition coefficients for both species are constants) may not be very realistic but are necessary simplifications. A variety of factors not included in the model can affect the outcome of competitive interactions by affecting the dynamics of one or both populations. Environmental change, disease, and chance are just a few of these factors.

# Additional Questions

1. The Lotka-Volterra model predicts that stable coexistence of two species is possible only when intraspecific competition has a greater effect than interspecific competition. Why would this be the case?

# Source

Begon, M., J. L. Harper, and C. R. Townsend. 1996. *Ecology: Individuals, Populations, and Communities, 3rd edition*. Blackwell Science Ltd. Cambridge, MA.

Gotelli, N. J. 1998. *A Primer of Ecology, 2nd edition*. Sinauer Associates, Inc. Sunderland, MA.

# About this Resource

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It is published using the QUBES Open Education Resources publishing platform[[4]](#endnote-4).

1. http://www.tiem.utk.edu/~gross/bioed/ [↑](#endnote-ref-1)
2. https://qubeshub.org/community/groups/quantbioatcc/ [↑](#endnote-ref-2)
3. https://openstax.org/details/books/biology-2e [↑](#endnote-ref-3)
4. https://qubeshub.org/qubesresources/publications/1061/ [↑](#endnote-ref-4)