State Apportionment Day 2

Knowledge of Students:
Students will be heterogeneously grouped.
Students should be familiar with percentages and ratios.

Learning Goals:
By the end of the lesson, the students will be able to…

- Make sense of the task and persevere in solving.
- Apply weighted averages to state apportionment.
- Recognize that none of the methods work perfectly.
- Use data about the actual state apportionment results to reinforce that apportionment is based on population.

Associated Standards:

A-SSE: Interpret the structure of expressions
1. Interpret expressions that represent a quantity in terms of its context.

SMP's: 1, 3, 4, 6, 8

Equipment/Materials:

- Chrome book
- State Apportionment Worksheet 1
- State Apportionment Worksheet 2

Associated Files & Websites:

- Lesson Plan State Apportionment
- Mathematical Modeling Handbook State Apportionment Task Handouts

http://www.cut-the-knot.org/ctk/Democracy.shtml
http://www.census.gov/history/www/programs/demographic/methods_of_apportionment.html
http://www.census.gov/population/apportionment/about/index.html
http://www.ctl.ua.edu/math103/apportionment/appmeth.htm
http://www.math.colostate.edu/~spriggs/m130/apportionment2.pdf
https://epubs.siam.org/doi/abs/10.1137/1020040

https://www.youtube.com/watch?v=YWfEqWLz9pc (Hamilton Method)
https://www.youtube.com/watch?v=weGGVmy9yLc (Jefferson Method)
https://www.youtube.com/watch?v=l74j-auLjZE (Huntington-Hill Method)
https://www.youtube.com/watch?v=_UdGUULKN-E (Geometric Mean)

Associated Text:

Task adapted from Sanfratello, A. (2012). State apportionment. In H., Gould, D.R., Murray, & A., Sanfratello (Eds.), *Mathematical modeling handbook* (pp. 133 – 140). Bedford, MA: The Consortium for Mathematics and Its Applications.

Students' Learning Activities, Teacher's Questions and Anticipated Student Responses	**Teacher's Support**	**Notes/Reflection Include hypothesis to try out in the future.**
Launch/Warm Up (15 minutes) Find the arithmetic mean and the geometric mean of each pair of numbers. Round to 2 decimal places when needed. What do you notice about how the two values compare to each other? Valid Response The geometric mean is always slightly smaller than the arithmetic mean. Emerging Responses The numbers are similar. If you round the geometric mean, you get the arithmetic mean.	Check that for all of the numbers. Is that true?	

	Arithmetic Mean	Geometric Mean
6 and 12	$(6 + 12)/2 = 18/2 = 9$	$\sqrt{(6\ x\ 12)} = 8.49$
5 and 9	$(5 + 9)/2 = 14/2 = 7$	$\sqrt{(5\ x\ 9)} = 6.71$
4 and 7	$(4 + 7)/2 = 11/2 = 5.5$	$\sqrt{(4\ x\ 7)} = 5.29$

Exploration/Group Work (30 - 35 minutes) Part 1

1. Watch the video on the Jefferson method, apply the Jefferson method for the 25 representatives.

Link: https://tinyurl.com/SGJefferson

Use the tables to apply the Jefferson method (students are provided three tables on the worksheet).

State	Population	Calculation	Number of Seats
A	15,000		
B	17,000		
C	28,000		
D	40,000		

Why did Jefferson use this method?

What are the differences and similarities between the Jefferson Method and the Hamilton Method?

Valid Responses

The Jefferson Method involves modifying the divisor, d, which is calculated by taking the quotient of the total population and the number of seats. d is then decreased until the quotient of each state's population and the new d add up to the exact number of seats needed.

Let d = 4000 (100,000/25)

If students struggle, assign each group a different denominator to try. Discourage students from erasing any trials that do not work. Encourage students to have at least one table with a denominator that does not work and one table with a denominator that does work.

For Desmos, instruct students to enter one divisor that does not work and one divisor that does work.

What's the purpose of this method? What do you hope to accomplish? How do you know when to stop guessing?

State	Population	Calculation	Number of Seats
A	15,000	15,000/4,000 = 3.75	3
B	17,000	17,000/4,000 = 4.25	4
C	28,000	28,000/4,000 = 7	7
D	40,000	40,000/4,000 = 10	10

Using the same method, 3,900 still does not work.

Let d = 3,700

State	Population	Calculation	Number of Seats
A	15,000	15,000/3,700 = 4.05	4
B	17,000	17,000/3,700 = 4.59	4
C	28,000	28,000/3,700 = 7.56	7
D	40,000	40,000/3,700 = 10.81	10

Students explain why some columns worked and others did not work.

Emerging Responses

Students focus on the procedure without attaching any meaning to the output.

Students struggle with finding the denominator to use.
Students will use a denominator that is not helpful such as 4,001 or 3,999.
Students may not recognize that the denominator needs to be smaller instead of larger.

Share Out

First: Have one group share that chose a divisor that did not work
Second: Have a different group share a divisor that works.
Third: Have another group share a divisor that works.

Have each person in your group choose a different divisor. It does not matter what divisor you choose. The end goal is to find the number of seats.

This is a really complicated method. This is why people are still arguing

Rationale (relationship to learning goal(s)):
Students use a different method for state apportionment.

over state apportionment. There is no right or wrong way.

Exploration/Group Work (30 - 35 minutes) Part 2

2. The current method, Huntington-Hill, used by the US uses the geometric mean as the denominator and the state's population in the numerator. Does this method work well?

What the video in the link below and complete the table.
https://tinyurl.com/SGHHmethod
Valid Responses

If the quotient is greater than the geometric mean, give the number of seats equal to the upper quota.

For 25 seats

State	Population	Calculation				Number of Seats
		Quota	Lower Quota (n)	Upper Quota (n + 1)	Geometric Mean $\sqrt{(n(n+1))}$	
A	15,000	.15 x 25 = 3.75	3	4	$\sqrt{(3\ x\ 4)}$= 3.46	4
B	17,000	.17 x 25 = 4.25	4	5	$\sqrt{(4\ x\ 5)}$= 4.47	4
C	28,000	.28 x 25 = 7				7
D	40,000	.4 x 25 = 10				10

Pause the video periodically and ask students what they notice about the video.

What questions do you have right now? Discuss your thoughts and questions with your group.

For 17 seats

State	Population	Calculation				Number of Seats
		Quota	Lower Quota (n)	Upper Quota (n + 1)	Geometric Mean $\sqrt{(n(n+1))}$	
A	15,000	.15 x 17 = 2.55	2	3	$\sqrt{(2\ x\ 3)}$= 2.45	3
B	17,000	.17 x 17 = 2.89	2	3	$\sqrt{(2\ x\ 3)}$= 2.45	3
C	28,000	.28 x 17 = 4.76	4	5	$\sqrt{(4\ x\ 5)}$= 4.47	5
D	40,000	.4 x 17 = 6.8	6	7	$\sqrt{(6\ x\ 7)}$= 6.48	7

The H-H method does not work well with 17 seats. It leads to 18 total seats.

Emerging Responses

Student groups will get stuck at different points along the way.
Students will complete the calculations, but not make sense of what the numbers mean.
Students will struggle with applying the formula correctly.

If students finish early, you can have them continue to work on the 17 seats by changing the divisor until they get the right number of representatives per state.

How many seats are you trying to fill? Remind students to review the goal of the task.

Share Out of Exploration Part 2 (10 minutes)

Share Out 1 (Mid-summary)

Choose groups that did not complete the chart. Have them share sticking points and receive feedback from the rest of the class.

Whole-Class Share Out

Whole-class discussion about the groups' final conclusions.

Discuss the outcomes with your groups. What do you notice or wonder?

Final Exploration—Connecting to the Real World 2. Look up the actual state populations and their apportionments. https://en.wikipedia.org/wiki/United_States_congressional_apportionment How many representatives does Delaware have? How does Delaware compare to other states? What are your thoughts about the current state apportionments? What else jumps out at you? What do you notice? Explore the Wikipedia and pick out something that speaks to you. Valid Responses Students will notice some state have many more representatives. For example, California has 53 representatives and Delaware only has 1. Students will recognize other states that also only have one representative but are much larger in size. There was one time in history that Delaware had 2 representatives. New York had 45 representatives. Then it went down to 16. Students reference population density or other reasons for varying numbers representatives. Emerging Responses Big states only have one representative, they should have more.	You are going to explore a website. Be prepared to share out anything that speaks to you. Remind students that they can sort the chart towards the middle of the page. Why might the apportionments be different historically? [The populations changed, and the apportionment methods changed over the years. Delaware is an interesting example.] Washington, D.C. does not have any representatives because it is not a state. What are your thoughts about that?	
Rationale (relationship to learning goal(s)): These tasks support students in their mathematical discourse and productive struggle. Students have the opportunity to engage in an authentic mathematical modeling task.		

Summary of Lesson (Time) Go back to your rough-draft thoughts about apportionment.	How did your ideas about apportionment change?	
Homework (Time) TBD		

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