State Apportionment Day 1

Knowledge of Students:
Students will be heterogeneously grouped.
Students should be familiar with percentages and ratios.
Students should be familiar with arithmetic mean and geometric mean.

Learning Goals:
By the end of the lesson, the students will be able to…

- Make sense of the task and persevere in solving.
- Explain their mathematical thinking about their selected proportions or state apportionment methods.
- Apply weighted averages to state apportionment.
- Compare and contrast various methods of apportionment.

Associated Standards:

A-SSE: Interpret the structure of expressions
1. Interpret expressions that represent a quantity in terms of its context.

SMP's: 1, 3, 4, 6, 8

Equipment/Materials:

- Chrome book
- State Apportionment Worksheet 1
- State Apportionment Worksheet 2

Associated Files & Websites:

- Lesson Plan State Apportionment
- State Apportionment Maps in Google Drive Folder

https://www.census.gov/library/video/census_apportionment_machine.html (apportionment machine)
http://www.cut-the-knot.org/ctk/Democracy.shtml
http://www.census.gov/history/www/programs/demographic/methods_of_apportionment.html
http://www.census.gov/population/apportionment/about/index.html
https://www.census.gov/library/visualizations/2010/dec/2010-map.html (Maps)

http://www.ctl.ua.edu/math103/apportionment/appmeth.htm
http://www.math.colostate.edu/~spriggs/m130/apportionment2.pdf
https://epubs.siam.org/doi/abs/10.1137/1020040
https://www.youtube.com/watch?v=YWfEqWLz9pc (Hamilton Method)
https://www.youtube.com/watch?v=weGGVmy9yLc (Jefferson Method)

Associated Text:

Task adapted from Sanfratello, A. (2012). State apportionment. In H., Gould, D.R., Murray, & A., Sanfratello (Eds.), *Mathematical modeling handbook* (pp. 133 – 140). Bedford, MA: The Consortium for Mathematics and Its Applications.

Students' Learning Activities, Teacher's Questions and Anticipated Student Responses	**Teacher's Support**	**Notes/Reflection Include hypothesis to try out in the future.**
Launch/Warm Up (10 minutes) If you were in charge of determining how many representatives each state in the United States should have, what information would you need. How would you use that information? What obstacles do you think might be present? <u>Valid Responses</u> Students will tie the population to the number of representatives. I will calculate the percent population of each state. I will find the total number of congressional seats. I will use the average. I would make sure the states with a higher population have higher seats. Smaller states have fewer, larger states have more (with regards to population). 1 representative per 100,000 people <u>Emerging Responses</u> I will divide by the total number of states. States with a larger area should have the highest number of representatives. Students think they need to know political parties, or the conversation turns into a discussion about who to elect.	Once students have two or three valid responses, choose those students to share out. Can you provide an example from the map? How do you know how many seats should be allocated? Why do you think that method is fair? For example, should Delaware have the same number of representatives as California? Why or why not?	

I would not assign any representatives. We will base all elections on the popular vote. Students think the apportionments are not fair because states with more representatives have more influence. **Warm Up Share Out** After students share their thoughts: How would you define apportionment? Write down your rough draft thoughts. Watch video as a whole class https://www.census.gov/library/video/census_apportionment_machine.html <u>Rough Draft Thoughts on Apportionment Responses</u> Apportionment is how representatives are distributed throughout the 50 states. Apportionment means breaking up something into groups for a certain reason.	In this case, we would have to vote all the time. Representatives vote on various initiatives including legislation and appointments for offices throughout the year. What is your definition of fair? What is your idea for improving apportionment? Students share definitions of apportionment on an online discussion board.	
Rationale (relationship to learning goal(s)): This question sets the stage and gives students a visual of the varying sizes of states and populations. Students may begin thinking about population density.		
Exploration/Group Work (25 minutes) Part 1 1. A. If the new country plans on having 25 representatives in its House of Representatives, how many should each state receive? B. What if they plan to have only 17 representatives? How did you calculate how many representatives each state should receive? Did you use the same method for both 25 and 17 representatives? A. <u>Valid Responses</u>	Explain your calculation.	

Students use percentages to calculate the number of seats to be allocated.
Students round up or down to decide on the number of seats per state.
Students divided each state population by the divisor (4,000) instead of using percentages.
Students set up a proportion to find the number of seats.

With 25 representatives, the states should receive the following apportionment: A = 4; B = 4; C = 7; D = 10.

State	Population	Calculation	Number of Seats
A	15,000	0.15 x 25 = 3.74 or 15,000/4,000	4
B	17,000	0.17 x 25 = 4.25 or 17,000/4,000	4
C	28,000	0.28 x 25 = 7 or 28,000/4,000	7
D	40,000	0.4 x 25 = 10 or 40,000/4,000	10

One possible ratio that may occur: (25/(100,000/15,000))

Emerging Responses

Students may divide by four and round.
Students may not use percentages. They might just guess and check.

Students do not round correctly, or students' allocations may not sum to 25.

Students may use tallies to divide up the representatives.

Use a simple example (e.g., if we share a pizza and I pay $15 and you pay $5 how much of the pizza should you get to eat?)

How would you round the number of people?
Why are we not able to round all of the allocations appropriately?

Is this a fair allocation? Why or why not?

Can you elaborate on that?

B. Valid Responses

Same as above.

With 17 representatives, the states should receive the following apportionment: A = 2; B = 3; C = 5; D = 7.

Initially, students follow rounding rules and round up to find the total of seats is 18 (see table below). Then students round A down since it has a smaller population.

State	Population	Calculation	Number of Seats
A	15,000	0.15 x 17 = 2.55 or 15,000/5882	3
B	17,000	0.17 x 17 = 2.89 or 17,000/5882	3
C	28,000	0.28 x 17 = 4.76 or 28,000/5882	5
D	40,000	0.4 x 17 = 6.8 or 40,000/5882	7

Emerging Responses

Students do not round correctly, or students' allocations may not sum to 17. Students may round the divisor to 6,000.

2. Which states (if any) would disagree with the apportionment that you have created in each of these cases? Do both scenarios create the same problems? Can you create a method that is fair to all states in both cases? Describe how your method works and why you believe it to be fair.

Valid Responses

For 1A: A and B get the same number of representatives even though B has more people. Students should question the fairness of this allocation.

Monitor students and ask students to share out their responses using the document camera.
What do you notice about each method? What are the pros and cons of each method?
What are your revisions for your initial responses?

For 1B: Students will notice that for state A, you have to round down even though the number is 2.55. Emerging Responses Students explanations provide a weak explanation for why their method works. States A and B might complain because they don't get as many representatives as the states with higher populations. No states will disagree because it's based on population. **Share Out of Exploration Part 1 (10 minutes)** Student Response Sequencing First: Guess and check weighting method Second: Percentage method & rounded to get too many or too few representatives Final: Rounded and ended up with the right number of representatives (compare two groups that rounded differently)		
Exploration/Group Work (30 minutes) Part 2 3. The Hamilton Method was devised by Alexander Hamilton as a technique for fair apportionment. Investigate what the Hamilton Method was and if you agree or disagree with its fairness. Do either of your methods share any similarities with the Hamilton Method? Watch the video on the Hamilton Method: https://tinyurl.com/SGHamilton Valid Responses Students will research the Hamilton Method on their laptops. The Hamilton Method always gives the states with the highest remainder the extra seat(s).	How does this method make sense? Can you justify why Hamilton is rounding everyone down in the beginning? How does he decide who gets the extra representatives? After the explanation of the method you can pause the video and start to use the Hamilton method. You may use software to build in pauses and interject questions.	

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<td>One student noticed that Hamilton was not working, so he started to adjust the divisor to try to apportion the seats more fairly.

For 17 total seats the remainders are:
A = 2; B = 3; C = 5; D = 7
Since A has the lowest remainder, the number of seats for State A is rounded down and the other states are rounded up.

Emerging Responses
Students will round using traditional rounding rules.
Students will not be able to decide on the extra person.</td>
<td>Refer to the video: Why did they add the extra person in this one? How did they decide?</td>
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<td>Exploration Share Out Part 2 (10 – 15 minutes)
Choose various organization techniques to share.</td>
<td rowspan="2">Does anyone have anything else to add?</td>
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<td>Rationale (relationship to learning goal(s)):
This task provides opportunities for students to think about how state representatives can be allocated fairly. Students use percentages and proportions to decide how states can be represented fairly.</td>
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<td>Exploration/Group Work (10 - 15 minutes) Part 3
4. Suppose that 1000 people move from state B to state A. How would this affect the Hamilton Method with both 25 and 17 representatives? Is this fair? Why or why not? Does it make sense that moving 1000 would change the results?

Valid Responses
Under the new population, with 25 representatives, the states receive the following apportionments: A = 4; B = 4; C = 7; D = 10.
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<tr><th>State</th><th>Population</th><th>Calculation</th><th>Number of Seats</th></tr>
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<td>Split up calculating 25 and 17 within groups (e.g., two students calculate 25 and each calculate 17).</td>
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A	16,000	.16 x 25 = 4	4
B	16,000	.16 x 25 = 4	4
C	28,000	.28 x 25 = 7	7
D	40,000	.40 x 25 = 10	10

With 17 representatives, the states should receive the following apportionments: A = 2; B = 3; C = 5; D = 7.

State	Population	Calculation	Number of Seats
A	16,000	.16 x 17 = 2.72	3
B	16,000	.16 x 17 = 2.72	3
C	28,000	.28 x 17 = 4.76	5
D	40,000	.40 x 17 = 6.8	7

This method creates 18 seats, so A or B would need to be rounded down.
In the latter apportionment, states A and B have the same population but do not receive equal representation.

Emerging Responses

Students will not find the correct allocations.
Students will not find the total number or representatives.
Students choose arbitrary reason for which state (A or B) to assign more representatives. For example, a students was observed, allocated an extra seat to State B, based the order of her calculations.
Students did not make the connection that the Hamilton method did not work with 17 representatives because the decimals were the same.
Students may give the extra representative to State D since it has the highest remainder and A and B have the same remainder.

Do you think your reasoning is fair? Why?

Does it make sense that moving 1000 would change the results?

You may want to use an online sharing platform to display student responses.

Share Out of Exploration Part 3 (10 – 15 minutes) With your partner discuss what you notice and wonder. Have a whole-class discussion about student responses.		
Summary of Lesson (5 minutes) What are the advantages of the Hamilton method? What areas for improvement do you see? Exit Ticket with Different Examples Students Choose Which Method Was Used to emphasize the difference between truncating and rounding		
Homework (Time) Watch the video on the geometric mean Examples where they truncate versus round		
Rationale (relationship to learning goal(s)): Students watch a new method that will be discussed the following day.		

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