**Learning with All Your Senses:**

**Mathematical Manipulatives Enhance Student Comprehension of Biological Models**

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Please join us for a workshop focused on “Kinesthetic Learning.” We have found that by engaging students in building and manipulating three-dimensional models of biological phenomena and mathematical models of these phenomena that many students who struggle with abstract and symbolic mathematical applications to biology can participate in building better models that they feel some ownership for and can transfer their knowledge to other applications. We will begin with a gallery of various models that range from the ecological and epidemiological level to molecular models. We will have computers and three dimensional printers as well as some other hands on materials for participants to construct their own models throughout the week. Some of the mathematical models will focus on geometry, topology, probability, statistics, finite difference equations, cellular automata, and algebra. All examples are intended to be accessible to students with high school algebra and geometry (i. e., no calculus is required).

Wojdak: The use of three dimensional printing in undergraduate education is maturing beyond the "Look what we can do!" phase. Instructors across all the biological disciplines are leveraging this technology to visualize data, processes, and morphologies, to create models to illustrate, and to replicate rare or delicate specimens. Moreover, instructors are pairing 3D printing with best-practice pedagogical practices to open new doors for students. We will examine the landscape of exemplary projects emerging at the intersection of novel technology and effective pedagogy.

Jungck: How does a viral capsid self-assemble? While Olson, Tibbits, and colleagues demonstrated self-assembly of dodecahedral viral capsid models via 4D printing, most capsids are icosahedral. We used Dürer nets, Schlegel diagrams, and CAD software to design triangular pieces which self-assemble into a icosahedron. A crucial design problem was realizing the importance of convex properties of the surface of the 3D printed equilateral triangular pieces interacting with the concaveness of the interior of the vessel used to assemble the pieces into the final icosahedral shape. We also have been printing 3D radiolarian representations built upon our 3D X-ray nanotomography data, analyzing them with medial axial transforms, and digital dissection. Using our software: Ka-me: A Voronoi Image Analyzer we not only re-examine Haeckel’s illustrations, but compare them with the geometry and topology of actual specimens. A web site of our radiolarian work is at: (<https://spark.adobe.com/page/lm464/>). By sharing our data via an open science depository: MorphoSource, other investigators will be able to analyze the raw data and/or print their own 3D models based upon the 3D voxel data. The artist Bathsheba Grossman used our 3D file to build embedded crystal glass representations of our radiolaria.

Below are articles, websites, projects and such referenced in the workshop, or on related topics.

The following references are from work by John Jungck and his collaborators:

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7. Link to talk at NIMBioS: (a) 4D printing and self-assembly (b) 3D printing and nano-tomography <http://www.nimbios.org/announcements/sem_jungck>; <https://www.youtube.com/watch?v=kq2wdXMnNe0&feature=youtu.be>

Materials form others relevant to Jungck's work:

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2. Herman, Tim, Jennifer Morris, Shannon Colton, Ann Batiza, Michael Patrick, Margaret Franzen, and David S. Goodsell. (2006). "Tactile teaching: Exploring protein structure/function using physical models." *Biochemistry and Molecular Biology Education* 34 (4): 247-254. [Tactile\_teaching\_Exploring\_protein\_structure-Tim Herman]
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6. Gurnon, Daniel, Julian Voss-Andreae, and Jacob Stanley. (2013). "Integrating art and science in undergraduate education." *PLoS Biology* 11 (2): e1001491. (4 pages).
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8. Roberts, J. R., E. Hagedorn, P. Dillenburg, M. Patrick, and T. Herman. (2005). “Physical models enhance molecular 3D literacy in an introductory biochemistry course.” *Biochemistry and Molecular Biology Education* 33: 105-110.
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12. Ford, Simon, and Tim Minshall. (2019). "Invited review article: Where and how 3D printing is used in teaching and education." *Additive Manufacturing* 25: 131-150.
13. Loy, Jennifer. (2019). "3D Printing Interdisciplinary Learning for Complex Problems." In *Interdisciplinary and International Perspectives on 3D Printing in Education*, pp. 94-109. IGI Global,
14. Another chapter in it: Creating Tactile Graphs for Students With Visual Impairments: 3D Printing as Assistive Technology Terence W. Cavanaugh (University of North Florida, USA) and Nicholas P. Eastham (University of North Florida, USA).
15. Bain, Gordon A., John Yi, Mithra Beikmohamadi, Timothy M. Herman, and Michael A. Patrick. (2006). "Using physical models of biomolecular structures to teach concepts of biochemical structure and structure depiction in the introductory chemistry laboratory." *Journal of Chemical Education* 83 (9): 1322. [<https://doi.org/10.1021/ed083p1322>]
16. Ruths, Derek A., Edward S. Chen, and Leland Ellis. (2000). "Arbor 3D: an interactive environment for examining phylogenetic and taxonomic trees in multiple dimensions." *Bioinformatics* 16 (11): 1003-1009.
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3. Youtube: Arthur Olson: Visualizing Molecular Recognition and Self-Assembly https://www.youtube.com/watch?v=fKGulHzykBU&t=40s

[self assembling virus](file:///Users/johnjungck/Downloads/self%20assembling%20virus) <https://www.youtube.com/watch?v=X-8MP7g8XOE>

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4. TED Talk: <https://www.youtube.com/watch?v=0gMCZFHv9v8>

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6. Howell, Michelle E., Karin van Dijk, Christine S. Booth, Tomáš Helikar, Brian A. Couch, and Rebecca L. Roston. (2018). "Visualizing the invisible: A guide to designing, printing, and incorporating dynamic 3D molecular models to teach structure–function relationships." *Journal of microbiology & biology education* 19 (3): (3 pages).

**SAVE:** Investigating DNA supercoiling-3D printed-jmbe.pdf

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