| Real-World Context | Possible Math Tools |
| :--- | :--- |
| Students divide into groups, and each group designs a zoo: it must have | $3-5$ students: Multiplication, |
| more than one exhibit, and students must consider multiple sources of | data analysis, linear functions |
| cost. Students may brainstorm one of many different modeling problems: |  |
| - How much should we charge for admittance? |  |
| - How could we reduce an outstanding cost? |  |
| - How can we design a profitable zoo with limited space? |  |
| - If we went on a field trip to the zoo, could we learn which exhibits are |  |

## Relevant Common Core Standards:

## CCSS.MATH.CONTENT.3.OA.A. 1

Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$. Task: Use grid paper to plan out several exhibits of different sizes, and find their area using rectangles.

## CCSS.MATH.CONTENT.3.OA.A. 3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
Task: Build a space-efficient zoo, given minima for the dimensions of each exhibit.
CCSS.MATH.CONTENT.3.OA.D. 8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Task: Design a zoo within some initial budget, including the cost of land.

## CCSS.MATH.CONTENT.3.NBT.A. 2

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
Task: Calculate the cost of a zoo with several different exhibits.

## CCSS.MATH.CONTENT.3.MD.B. 3

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve oneand two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.
Task: Weigh the costs of different exhibits at different times.

## CCSS.MATH.CONTENT.3.MD.C. 6

Measure areas by counting unit squares (square cm , square m , square in, square ft , and improvised units). Task: Plan out a zoo with grid paper-define units which make sense, and note the area of each exhibit.

## CCSS.MATH.CONTENT.3.MD.D. 8

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Task: Put together several exhibits, first minimizing total area, then minimizing perimeter. Remember to leave walking paths.

## CCSS.MATH.CONTENT.5.NBT.B. 7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Task: Analyze the costs of a zoo exhibit, as well as how often they come up. Resolve daily, weekly, and yearly costs to a single formula.

CCSS.MATH.CONTENT.5.MD.C. 5
Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
Task: Represent an exhibit with 3-dimensional shapes, and find its cost (e.g. building an indoor display, digging a tiger moat).

CCSS.MATH.CONTENT.5.G.A. 2
Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Task: Represent each exhibit on the coordinate plane, and find important values such as walking distance.

Dive into Math Modeling

Dive In:
Students begin exploring the topic.

| Student Actions | Teacher Actions |
| :---: | :---: |
| Students will explore the topic by answering questions such as: <br> - What do you notice? What do you wonder? <br> - What is interesting about this topic? <br> - What about this topic is important? <br> - What information do you need? | What will you show/tell students to launch the real-world context and capture their interest? <br> - Discuss a visit to the zoo. What can be seen there? How many exhibits do students remember? What makes them different? <br> - Next, ask if students have any pets. What are the costs associated with owning a pet? Food? Cleanup? What about time commitment? |
| Students will brainstorm these questions in groups. | Allow students time to brainstorm. Monitor student progress and group dynamics. <br> Take note of anything that should be shared with the class: <br> - ideas that help students mathematize the problem <br> - common misconceptions |

Define the Problem:
Ideas are narrowed to a focused, mathematically relevant problem.

| Student Actions | Teacher Actions |
| :--- | :--- |
| $\begin{array}{l}\text { Students will choose a focused problem } \\ \text { that can be answered and justified with } \\ \text { information and mathematics. }\end{array}$ | $\begin{array}{l}\text { Guide students towards a focused problem that can be } \\ \text { answered and justified with information and mathematics. }\end{array}$ |
| $\begin{array}{l}\text { Students should consider questions such } \\ \text { as: }\end{array}$ | $\begin{array}{l}\text { What are my expectations for the model? Will the whole class } \\ \text { focus on the same problem, or will variation be allowed? } \\ \text { Different groups may have different ways of } \\ \text { - What information do you need to }\end{array}$ |
| $\quad \begin{array}{l}\text { make a model? }\end{array}$ |  |
| - What quantities are required by the cost. For example, one group may focus |  |
| on building and maintaining the habitats, while |  |
| another group may focus on providing food and |  |$\}$| other care. |
| :--- |

How will you guide your students to use new skills they are less comfortable with?

Encourage students to consider the cost of exhibits after some time (e.g. one week) using multiplication.

Do the Math:
Iterate the model until it is done and can be evaluated.

| Student Actions | Teacher Actions |
| :--- | :--- |
| Use mathematical tools to develop a <br> model. <br> Mathematically justify all estimations and <br> numerical values in model. <br> Use the model to suggest a solution. <br> Record work. | Note the mathematics that develops during model building. <br> What are some common misconceptions that could arise at <br> this stage, and how might you address them? <br> Some costs are only paid once, while others are daily, <br> weekly, or monthly; students may not remember <br> this. Ask students to consider what their zoo would <br> be like in a year: what would they still be paying? |
|  | Address misconceptions individually or as a group. |
| When are natural times to regroup? <br> After the students have completed drafts: they <br> should know what exhibits they want to include, <br> estimate their costs, and conduct surveys on which <br> ones are popular. |  |

## Decide Whether You're Satisfied, and Declare Victory:

Evaluate your model and decide when the model is ready to be presented.

| Student Actions | Teacher Actions |
| :--- | :--- |
| Students should be evaluating their model <br> by asking questions such as: <br> - If there is a rubric or checklist, see if <br> you did everything. <br> - Is your solution reasonable? Why or <br> why not? | What components do you expect the students' models to <br> include? |
| A map of the zoo, a table of costs, and a suggested |  |
| price of admission. |  |

## Demonstrate Solution:

Present and interpret your model that solves the problem.

| Student Actions | Teacher Actions |
| :--- | :---: |
| Students will reflect, justify, and present | What expectations do you have for students' presentations? |
| their models by asking and answering | Groups should be able to properly explain the |
| questions such as: | decisions they made, and how they relate to the <br>  <br> mathematical topics of interest. |

- Why would you recommend your model to someone?
- What mathematical tools did you use, and how did they help solve the problem?
- What did you change in your model throughout the modeling process?
- Are there situations where your solution wouldn't work or your model wouldn't apply?
- How would you need to change your model to apply to more situations?
- If you had more time, what else would you do?
- Are there any mathematical tools or pieces of information that would have been helpful to have?

MUDD
Dive into Math Modeling!

## Revisit:

These questions may help you consider possible extensions to the problem. Tying the problem to more advanced math gives students a frame of reference for newer mathematical tools.

Q: When could you recall the math used in this lesson as a starting point or an example later in your curriculum?
A: This is problem could apply to any model where "initial costs" and "daily costs" must both be considered. More complicated modeling problems may refer to this one as an example of the necessary methods.

Q: Is there a time later in the year when you might come back to this real-world scenario with different mathematical tools? Remember that students sometimes reach for tools that are most familiar and it might take them a while to build confidence to use a new tool in a modeling situation.
A: As students learn more linear algebra, they can discover new techniques for predicting and optimizing cost.

Q: $\quad$ Throughout the year, will you be collecting new information about this scenario? Are there times you could use that information to reflect on and improve your model?
A: Information should be collected in a single day.

Q: Are there other similar scenarios where you could use the same kinds of models? What might change? What might stay the same?
A: Replace "zoo" with "shopping center," "museum," or one of many other businesses. The same methods may be used, but the sources of cost should work differently, perhaps even nonlinearly.

For more resources on how to change parameters and constraints or how to extend this task to other grades, consider consulting the GAIMME report pages 136-139 http://www.siam.org/reports/gaimme.php.

Dive into Math Modeling!

