Lab Assignment 5

Stochastic Acquatic Succession

Written by

Prof. Erin N. Bodine

Prepared for

Math 214: Discrete Math Modeling (Fall 2014)

Rhodes College

Pre Lab Assignment Due: Tuesday, 10/7/2014 before class begins Lab Assignment Due: Thursday, 10/9/2014 before class begins

Material from the prelab is covered on the quiz on 10/7/2014.

Recall from our reference readings the example of ecological succession of coastal wetlands. In this example, we consider 100 hectares of wetlands, which at the initial time are always submerged. Every decade, 5% of the always submerged wetlands become seasonally submerged, and 12% of the seasonally submerged wetlands becomes always dry. All hectares that are dry remain dry.

In this lab we will learn how to use MATLAB to simulate a matrix model representing this example of ecological succession, as well as a few other examples. We will also examine what happens when rate of movement from one state to another is stochastic.

In this lab, we will be using MATLAB to help achieve these goals. Thus, as a part of this lab you will learn how to do the following in MATLAB:

- Enter vectors in Matlab in either row form or column form.
- Enter matrices in Matlab.
- Simulate a matrix model.
- Print output tables in the Command Window.
- Plot the dynamics of a matrix model over time.

MATLAB Skills

At this points, you already know how to do quite a bit in MATLAB, however there are a couple of new MATLAB skills you will need in order to complete this lab. As a part of the Prelab Assignment, read through the following sections. Note, all examples files are posted on Moodle.

Vectors in Matlab

Let

$$\mathbf{x} = \begin{bmatrix} 2 & 5 & 2 & 4 & 6 \end{bmatrix}$$
 and $\mathbf{y} = \begin{bmatrix} 4 \\ 7 \\ 5 \\ 8 \\ 11 \end{bmatrix}$.

Since \mathbf{x} is represented as one row with many columns, we call \mathbf{x} a $row\ vector$, and since \mathbf{y} is represented as one column with many rows, we call \mathbf{y} a $column\ vector$. Note, that both vectors have the same number of elements. To enter these two vectors in to MATLAB we would use the commands

$$x = [2 \ 5 \ 2 \ 4 \ 6]$$
 and $y = [4; \ 7; \ 5; \ 8; \ 11]$

The semicolon indicates that the next entry should be placed in the next row of the vector. Try entering these two commands into the command window of MATLAB. Do your vectors appear as they should?

If you want x to appear as a column vector, enter x' into the command window. This creates what is called the transpose of the vector x, which has the effect of transforming a row vector into a column vector. If you enter y' into the command window, it will transpose the column vector y into a row vector.

Matrices in Matlab

The name MATLAB is actually short for Matrix Laboratory. One of Matlab's main features is that is quickly and easily handles and manipulates matrices.

1. The ecological succession of coastal wetlands described at the beginning of this lab can be modeled with the matrix equation,

$$\mathbf{x}(t+1) = \begin{bmatrix} 0.95 & 0 & 0 \\ 0.05 & 0.88 & 0 \\ 0 & 0.12 & 1 \end{bmatrix} \mathbf{x}(t), \text{ where } \mathbf{x}(t) = \begin{bmatrix} a(t) \\ s(t) \\ d(t) \end{bmatrix}$$

and $\mathbf{x}(t)$ represents the structure of the coastal wetlands at time step t, where a(t) is the proportion of wetlands that is always submerged at time step t, s(t) is the proportion that is seasonally submerged at time step t, and d(t) is the proportion that is always dry at time step t. Let

$$\mathbf{T} = \begin{bmatrix} 0.95 & 0 & 0 \\ 0.05 & 0.88 & 0 \\ 0 & 0.12 & 1 \end{bmatrix}.$$

To enter T into Matlab we would use the command

```
T = [0.95 \ 0 \ 0; \ 0.05 \ 0.88 \ 0; \ 0 \ 0.12 \ 1]
```

Note that the semicolon is again used to denote when to start the next row of entries. Enter the command into the command window and press Enter. Notice that the output displays as a matrix.

- 2. Suppose we want to retrieve the element of **T** in the third row, second column. To do this we use the command T(3,2). Enter this command into the command window and press Enter. In general, if we want to retrieve the element in the i^{th} row and j^{th} column we use the command T(i,j). Try retrieving the element in the second row, first column.
- 3. What if you want to retrieve and entire column or row? If you wish to retrieve the i^{th} row, use the command T(i,:), where the colon indicates all columns. If you wish to retrieve the j^{th} column, use the command T(:,j). Here, the colon indicates all rows.

Simulate a Matrix Model

1. From Moodle, download the script file LO5Ex01.m.

```
L05Ex01.m

% Lab 5 Example 1 by Prof Bodine
% Simulates Coastal Wetlands Ecological Succession Model
clear all

% Transfer matrix
T = [0.95 0 0; 0.05 0.88 0; 0 0.12 1];
% Initial condition: a(0) = 1, s(0) = 0, d(0) = 0

x0 = [1; 0; 0];
```

```
% Generate solutions over 100 times steps
x(:,1) = x0;
for n = 1:100
x(:,n+1) = T * x(:,n);
end
```

- 2. Now, let us take a moment to understand everything within this script.
 - (a) The first three lines give a description of the file in comments and use the clear command to erase the values of all variables and parameters from the workspace. If you do not remember how this command works, please review Lab 2, page 3.
 - (b) Line 6 defines the matrix T, and line 8 defines the initial state of the coastal wetlands (all of the 100 hectares are always submerged and thus in the first state, a).
 - (c) In this code we generate solutions over 100 times steps. The solution at each time step gives the proportion of the coastal wetlands in each state (a, s, and d) at that time step, and is represented as a column vector

$$\mathbf{x}(t) = \begin{bmatrix} a(t) \\ s(t) \\ d(t) \end{bmatrix}.$$

In Matlab, we save each of these column vectors as one column in a matrix called x. In line 11, we save the initial condition as the first column of x. Then in lines 12–14, we use a for-loop to generate the solutions at time steps 1 through 100. Each pass through the for-loop generates the solution at time step n, and saves that solution in the n+1 column of the matrix x. Notice, x(:,n) indicates that all rows in column n of matrix x are being retrieved.

- (d) Take another moment to review LO5ExO1.m. Do you understand what each line of code is doing? If not, now is the time to stop and ask.
- 3. Run the script file. To see what the solutions are, double click on the x variable listed in the Workspace window of Matlab. You should see a window open that look like Figure 1. You can scroll to the right to see view more. Remember, each column of x represents the "structure" of the coastal wetlands where the first row gives the proportion which is always submerged, the second row gives the proportion which is seasonally submerged, and the third row gives the proportion which is always dry.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
|---|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|
| 1 | 1 | 0.9500 | 0.9025 | 0.8574 | 0.8145 | 0.7738 | 0.7351 | 0.6983 | 0.6634 | 0.6302 | 0.5987 | 0.5688 | _ |
| 2 | 0 | 0.0500 | 0.0915 | 0.1256 | 0.1534 | 0.1757 | 0.1933 | 0.2069 | 0.2170 | 0.2241 | 0.2287 | 0.2312 | |
| 3 | 0 | 0 | 0.0060 | 0.0170 | 0.0321 | 0.0505 | 0.0716 | 0.0948 | 0.1196 | 0.1456 | 0.1725 | 0.2000 | |
| Λ | ← III | | | | İ | | | | | | | ĺ | F T |

Figure 1: Example of window that appears when you double click on the x variables listed in the Workspace window of MATLAB.

Creating Output Tables using fprintf & a For-Loop

Viewing the elements of the matrix x from the Workspace window can sometimes be a bit overwhelming if the matrix you are viewing is quite large. Sometimes, it is nice to view just a sampling of the matrix. We can print out the solutions at only the time steps we are interested in by using an fprintf statement inside a for-loop.

1. From Moodle, download the script file LO5ExO2.m.

```
____ L05Ex02.m -
      % Lab 5 Example 2 by Prof Bodine
1
      % Simulates Coastal Wetlands Ecological Succession Model
2
      clear all
3
4
      % Transfer matrix
      T = [0.95 \ 0 \ 0; \ 0.05 \ 0.88 \ 0; \ 0 \ 0.12 \ 1];
6
      % Initial condition: a(0) = 1, s(0) = 0, d(0) = 0
      x0 = [1; 0; 0];
8
9
      % Generate solutions over 100 times steps
10
      x(:,1) = x0;
11
      for n = 1:100
12
13
         x(:,n+1) = T * x(:,n);
      end
14
15
      % Print table of solution for a selection of time steps
16
17
      fprintf(' t
                         a
                                S
                                       d\n')
      fprintf('----\n')
18
      for n = [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 10 \ 20 \ 30 \ 40 \ 50 \ 75 \ 100]
19
           fprintf(\frac{1}{3}d \frac{4.3f}{4.3f} \frac{4.3f}{4.3f}, \frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \frac{1}{n}
20
      end
21
```

- 2. Now, let us take a moment to understand everything within this script.
 - (a) First, note that lines 1-14 are copied from the script file LO5Ex01.m.
 - (b) In lines 16–21 we print out a table of the solution for selected time steps. Lines 17 and 18 print out a header for the table, and lines 19–21 use a for-loop to fill in the table. Each pass through the for-loop fills in one line of the table, which represents the solution for one time step. Notice, that we are not printing the solutions at all time steps, but only for a selection of time steps given in the list

```
n = [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 10 \ 20 \ 30 \ 40 \ 50 \ 75 \ 100]
```

in the set up of the for-loop.

(c) In the fprintf statement on line 20, the %3d command is used to hold 3 spaces for the integer value of n, and %4.3f is used to hold 5 spaces (one digit before the decimal, a decimal, and three digits after the

- decimal) for the floating point values (real number values) of x(1,n+1), x(2,n+1), and x(3,n+1). Notice, these retrieve the three values in the n+1 column of x which represents time step n.
- (d) Take another moment to review LO5ExO1.m. Do you understand what each line of code is doing? If not, now is the time to stop and ask.
- 3. Run the script file. You should see some output printed in the command window (see Figure 3 for reference). What is happening to the structure of the coastal wetlands over time?

| Command Window | | | | | | | | |
|----------------|------------|-------|-------|-------|--|--|--|--|
| | >> L05Ex02 | | | | | | | |
| | t | a | s | d | | | | |
| | | | | | | | | |
| | 0 | 1.000 | 0.000 | 0.000 | | | | |
| | 1 | 0.950 | 0.050 | 0.000 | | | | |
| | 2 | 0.902 | 0.091 | 0.006 | | | | |
| | 3 | 0.857 | 0.126 | 0.017 | | | | |
| | 4 | 0.815 | 0.153 | 0.032 | | | | |
| | 5 | 0.774 | 0.176 | 0.050 | | | | |
| | 10 | 0.599 | 0.229 | 0.173 | | | | |
| | 20 | 0.358 | 0.201 | 0.441 | | | | |
| | 30 | 0.215 | 0.138 | 0.647 | | | | |
| | 40 | 0.129 | 0.087 | 0.784 | | | | |
| | 50 | 0.077 | 0.054 | 0.869 | | | | |
| | 75 | 0.021 | 0.015 | 0.963 | | | | |
| | 100 | 0.006 | 0.004 | 0.990 | | | | |
| fx | >> | | | | | | | |

Figure 2: Output printed in the Command Window when the script file LO5ExO2.m is run.

Generating Plots of Matrix Model Dynamics over Time

A picture says a 1000 words. Well, a graph of our solutions might tell us more about

1. From Moodle, download the script file LO5ExO3.m.

```
x(:,1) = x0;
11
      for n = 1:100
12
         x(:,n+1) = T * x(:,n);
13
      end
14
15
      % Plot solutions over time
16
      plot(0:100,x)
17
      xlabel('Time (decades)'); ylabel('Proportion of 100 hectares');
18
      legend('a','s','d','Location','East')
19
```

- 2. Now, let us take a moment to understand everything within this script.
 - (a) First, note that lines 1-14 are copied from the script file LO5Ex01.m.
 - (b) Lines 16–19 create a graph with three curves: one curve is the proportion of wetlands always submerged over time, the second is the proportion that is seasonally submerged over time, and the third is the proportion that is dry over time. Notice, that the plot command in the script does not explicitly list the three curves, nor does it define the color for each curve.
 - (c) Line 18 produces axes labels for the graph.
 - (d) Line 19 produces a legend for the graph and places it in the middle, right portion of the graph. Other possible locations are given in Table 1.
 - (e) Take another moment to review LO5ExO1.m. Do you understand what each line of code is doing? If not, now is the time to stop and ask.

| Specifier | Location in Axes | Specifier | Location in Axes | |
|--------------------------|---|--------------------------|---------------------------------|--|
| North | Inside plot box near top | South | Inside bottom | |
| East | Inside right | West | Inside left | |
| NorthEast | Inside top right (default for 2-D plots) | NorthWest | Inside top left | |
| SouthEast | Inside bottom right | SouthWest | Inside bottom left | |
| NorthOutside | Outside plot box near top | SouthOutside | Outside bottom | |
| EastOutside | Outside right | WestOutside | Outside left | |
| ${\bf NorthEastOutside}$ | Outside top right (default for 3-D plots) | ${\bf NorthWestOutside}$ | Outside top left | |
| SouthEastOutside | Outside bottom right | ${\bf SouthWestOutside}$ | Outside bottom left | |
| Best | Least conflict with data in plot | BestOutside | Least unused space outside plot | |

Table 1: Possible locations to place a legend in a Matlab plot.

3. Run the script file. You should see some output printed in the command window (see Figure 3 for reference). What is happening to the structure of the coastal wetlands over time? You should observe, that by time step 100 (100 decades = 10,000 years), almost all of the wetlands are completely dry land.

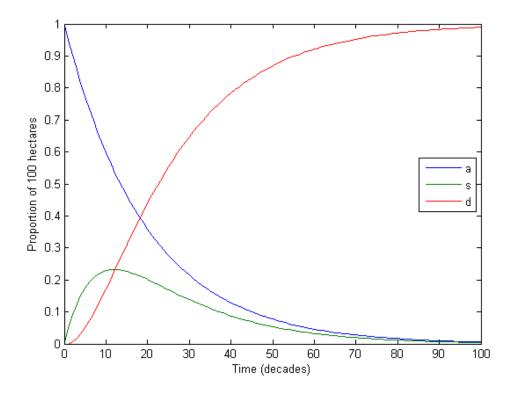


Figure 3: Plot produced from L05Ex03.m script.

PreLab Assignment

Recall in Example 2 of the reference readings we constructed a model of the ecological succession of coastal wetlands where the dynamics were a bit more complex (see Figure 4). In this prelab, you will create a MATLAB script which simulates the matrix model describing this wetlands succession, prints out the solution of the model at various time steps, and generates a graph of the solutions over time.

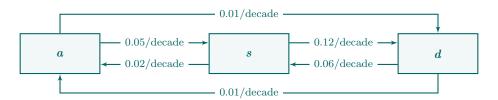


Figure 4: Flow diagram describing how the structure of coastal wetlands change over time.

1. Write a MATLAB script which generates solutions for the model described by the flow diagram in Figure 4 over 200 decades, prints out the structure of the coastal wetlands at time steps 0, 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100, and 200, and generates a graph of the solutions over the 200 decades.

The print out the of the structure of the coastal wetlands should have the appearance of a table with a header. Each row should correspond to one time step, and columns should correspond to the time step, a(t), s(t), and d(t).

The graph should have clear axes labels and and a legend placed in a location where it does not overlap with the plotted curves.

Name your script file PLabO5_YourLastName.m.

Make sure you script file is running without error and producing the output you expect before continuing to the next step.

- 2. In the command window type diary Plab05_YourlastName.out and press Enter. Run you script file, then type into the command window diary off and press Enter. Open up the diary file and make sure it has recorded the output of your script file.
- 3. Save the graph generated from your script file as PLabO5_YourLastName.jpg.
- 4. Submit your script file, diary file, and image file to Moodle. In the text box on Moodle, describe what happens to the structure of the coastal wetlands over time in this model. Compare the results to the simpler model explored in the MATLAB skills section of the PreLab.

Lab Assignment

For your lab assignment you will continue working with the model from the PreLab Assignment, and add a stochastic element to the model. Suppose the rate at which always submerged wetlands became seasonally submerged wetlands varied between 0.01/decade and 0.08/decade. How will this change what happens in the model.

If you do not recall how to generate random number in MATLAB, please review Lab 3.

- 1. Modify the MATLAB script you created for the prelab so that the rate at which always submerged wetlands become seasonally submerged wetlands is sampled from U[0.01, 0.08]. The rate should be re-sampled at each time step, which means the matrix must be redefined at each time step (i.e. within the for-loop). Remember, that each column in the matrix T must sum to 1. Name your script file Lab05_YourLastName.m.
 - Make sure you script file is running without error and producing the output you expect before continuing to the next step.
- 2. In the command window type diary LabO5_YourLastName.out and press Enter. Run you script file 3 times. Each time you run your script, save the resulting figure using the names LabO5a_YourLastName.jpg, LabO5b_YourLastName. and LabO5c_YourLastName.jpg. Next, type into the command window diary off and press Enter. Open up the diary file and make sure it has recorded the output of your script file.

3. Submit:

- 1 script file (LabO5_YourLastName.m)
- 1 diary file (Lab05_YourLastName.out)
- 3 image files (Lab05a_YourLastName.jpg, Lab05b_YourLastName.jpg, and Lab05c_YourLastName.jpg)
- In the text box on Moodle, describe how adding the stochastic element changed the output of the model. Is there a little variation or a lot of variation? Would you feel comfortable making predictions about the structure of the coastal wetlands after many years using this stochastic version of the model? Explain why or why not.