Lab Assignment 6

Equilibrium Structure in an Aquatic Succession Model

Written by

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Prepared for

Math 214: Discrete Math Modeling (Fall 2014)

Rhodes College

PreLab Assignment Due: There is no pre-lab assignment for this lab

Lab Assignment Due: Friday, 10/17/2014 by 11:59 pm

Recall the coastal wetland succession model we worked with in Lab 5. The ecological succession of coastal wetlands can be modeled with the matrix equation,

$$\mathbf{x}(t+1) = \begin{bmatrix} 0.95 & 0 & 0 \\ 0.05 & 0.88 & 0 \\ 0 & 0.12 & 1 \end{bmatrix} \mathbf{x}(t), \text{ where } \mathbf{x}(t) = \begin{bmatrix} a(t) \\ s(t) \\ d(t) \end{bmatrix}$$

and $\mathbf{x}(t)$ represents the structure of the coastal wetlands at time step t, where $a(t)$ is the proportion of wetlands that is always submerged at time step t, $s(t)$ is the proportion that is seasonally submerged at time step t, and $d(t)$ is the proportion that is always dry at time step t. Recall, that from simulations we determined that the equilibrium structure of the coastal wetlands using this model is $a = 0$, $s = 0$, and $d = 1$, which can be represented as the vector

$$\mathbf{x}^* = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We can use matrix algebra to determine the normalized eigenvector representing the equilibrium structure (as we did in the reference readings), but we can also use MATLAB to compute this vector. In the MATLAB Skills portion of this lab you will learn how to utilize MATLAB to compute eigenvectors, and how normalize a vector.

MATLAB Skills

Calculating Eigenvalues & Eigenvectors

1. Open MATLAB and navigate the current directory to the location where you will save your work.
2. In the command window, save the matrix
$$\begin{bmatrix} 0.95 & 0 & 0 \\ 0.05 & 0.88 & 0 \\ 0 & 0.12 & 1 \end{bmatrix}$$
as the variable `T`. If you do not remember how to do this, please review Lab 5.
3. To compute the eigenvector of `T` corresponding to the equilibrium structure use the command

```
[V,D]=eigs(T,1)
```

An $n \times n$ matrix actually has n eigenvectors (and n eigenvalues). We will learn more about this when we go through Section 5 of the reference readings. Each eigenvalues is associated with a particular eigenvector, but the eigenvector corresponding to the equilibrium structure of a matrix model is the eigenvector associated with the dominant eigenvalue (which is defined as the eigenvalue with the largest absolute value). We will have a detailed discussion about eigenvalues and eigenvectors when we go through Section 5 of the reference readings.

In MATLAB , the command `D = eigs(T,1)` computes the dominant eigenvalue of the matrix `T`. If you additionally wish MATLAB to return the eigenvector associated with the dominant eigenvalue you must use the command `[V,D] = eigs(T,1)`, where `D` is the dominant eigenvalue, and `V` is the eigenvector associated with the dominant eigenvalue.

Command Window

```
>> T = [0.95 0 0; 0.05 0.88 0; 0 0.12 1]

T =
    0.9500         0         0
    0.0500    0.8800         0
         0    0.1200    1.0000

>> D = eigs(T,1)

D =
    1.0000

>> [V,D] = eigs(T,1)

V =
   -0.0000
   -0.0000
   -1.0000


D =
    1.0000
```

4. When you execute the command `[V,D] = eigs(T,1)`, notice that the dominant eigenvalue is 1; this will always be the case for transfer matrices (i.e. matrices where the sum of each column in the matrix is 1). Additionally, notice that the eigenvector is

$$\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

At first glance this appears to be incorrect, but if we normalize this vector, it becomes

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Recall, that the normalized version of a vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

is one where each element of $\mathbf{x}$ has been scaled so that all the elements of $\mathbf{x}$ sum to 1. Thus, is $s = x_1+x_2+\cdots+x_n$

(the sum of all the elements of **x**), then the normalized version of **x** is

$$\begin{bmatrix} x_1/s \\ x_2/s \\ \vdots \\ x_n/s \end{bmatrix},$$

i.e., each element of **x** has been divided (or scaled) by the sum of the elements of **x**. To do this in MATLAB , we use the command

```
normx = x ./ sum(x)
```

Notice we use the `./` operator which divides each element of `x` by `sum(x)`, and the MATLAB function `sum(x)` takes the sum of all the elements in the vector `x`.

Try computing the normalized eigenvector corresponding to the equilibrium structure of the matrix **T**. If you do not get

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

now is the time to stop and ask for help.

Command Window

```
>> normV = V ./ sum(V)

normV =
    0.0000
    0.0000
    1.0000
```

Lab Assignment

Recall in Example 2 of the reference readings we constructed a model of the ecological succession of coastal wetlands where the dynamics were a bit more complex (see Figure 1). In this lab assignment, you will explore how the equilibrium structure of the coastal wetlands change as you change various parameters in this model

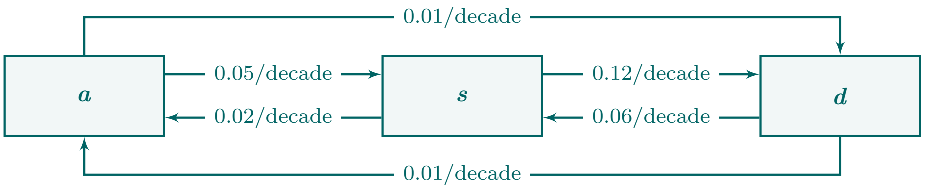

Figure 1: Flow diagram describing how the structure of coastal wetlands change over time.

1. In the command window type `diary Lab06_YourLastName.out` and press Enter. Then, (still in the command window) save the matrix corresponding to the flow chart in Figure 1 as `T`, and compute the normalized eigen-

vector corresponding to the equilibrium structure of the model. Lastly, (still in the command window), type `diary off` and press Enter. Open up the diary file and make sure it has recorded what you did in the command window. If not, try again.

2. Construct a script file in MATLAB , saved as `Lab06_YourLastName.m`, which accomplishes the following tasks in the order given:
    - (a) Erase the values of all variables and parameters stored in the workspace. If you do not remember how to do this, please review one of the previous labs.
    - (b) Create an array of values saved as the variable `r`, where $r = 0, 0.01, 0.02, \ldots, 0.70$.
    - (c) Use a for-loop over the values $1, 2, \ldots,$ `length(r)` to
        - (i) Construct the matrix according to the flow diagram in Figure 2. Save the matrix as the variable `T`.
        - (ii) Compute the normalized eigenvector of `T` corresponding to the equilibrium structure of the model.
        - (iii) Save normalized eigenvector computed in step (ii) in a matrix called `EQ`. Each column of `EQ` should correspond to one normalized eigenvector, and each pass through the loop should add one column to `EQ`. If you are unsure of how to do this, pleas review Lab 5.

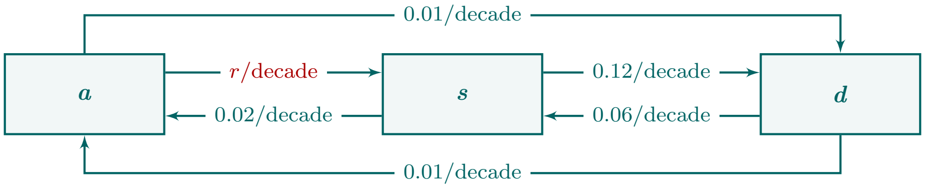

Figure 2: Flow diagram describing how the structure of coastal wetlands change over time. Note that the rate of flow from a to s is r per decade where $r = 0, 0.01, 0.02, \ldots, 0.70$.

    - (d) Create a plot showing how the equilibrium structure changes as the value of r changes. You graph should have three curves (one for each state in the model). Change the range of the vertical axis to the interval $[0, 1]$. If you do not remember how to do this, please review `L03Ex01.m` in Lab 3.

      Additionally, make sure your graph has appropriate axes labels, and a legend which indicates what each curve represents and is in a location that does not overlap with any of the curves.

3. Run your script file and save the graphical output as `Lab06_YourLastName.jpg`.

4. **Submit:**
    - 1 diary file (`Lab06_YourLastName.out`)
    - 1 script file (`Lab06_YourLastName.m`)
    - 1 image file (`Lab06_YourLastName.jpg`)
    - In the text box on Moodle, describe how changing the value of r changes the equilibrium structure? Suppose you increase the value of r by 0.01. Will this cause a greater change in the equilibrium structure if you increase from 0.02 to 0.03, or if you increase from 0.52 to 0.53? Explain by referencing your graph.