Sample solutions for the exercises
Exploring Erdős-Rényi random graphs with IONTW

Sample solution for Exercise 1: In some runs, almost no secondary infections occur, which indicates that the connected component of j^* is very small. In other runs, more than half of all nodes get infected. This indicates that j^* belongs to a large component that comprises more than half (for a typical instance, around 58%) of all nodes. It must be the same component each time, since different connected components must be disjoint and there cannot be 2 distinct ones that contain more than half of all nodes. □

Sample solution for Exercise 2: The results are similar to those of the previous exercise in that we observe a the same dichotomy between very small connected components and one very large component. However, the relative size of the large component now fluctuates around 0.58. The reason is that we create a new instant of $G_{ER}(300, 1.5)$ for each run. □

Sample solution for Exercise 3: In our experiment we observed a distinct gap between 34 runs where j^* belonged to a component of size at most 10 (a proportion of at most 0.03 of all nodes) and 66 runs where j^* belonged to a component of size at least 115 (a proportion of at least 0.38 of all nodes). The mean size of these large components was 174.1, which represents a proportion of 0.58 of all nodes. These results confirmed our observation in the previous exercises. □

Sample solution for Exercise 4: (a) Yes, the results of Exercise 3 were remarkably close to the theoretical prediction.

(b) In our experiment with 100 runs for $G_{ER}(300, 2)$ we observed 20 runs where j^* belonged to a component of size at most 4 (a proportion of at most 0.013 of all nodes) and 80 runs where j^* belonged to a component of size at least 206 (a proportion of at least 0.69 of all nodes). The mean size of these large components was 239.15, which represents a proportion of 0.7972 of all nodes.

In our experiment with 100 runs for $G_{ER}(300, 0.5)$ we did not observe a distinct gap between small and large components. In 64 runs, j^* was an isolated node. The largest connected component of j^* that we found had size 10, which represents a proportion of 0.033 of all nodes.

These results were remarkably close to the theoretical predictions. □

Sample solution for Exercise 5: (b) A given edge $\{i, j\}$ will be considered only at the first time step t when either node i or node j is infectious. Since we are assuming an SIR-model, after this time step, at least one of the nodes i, j will be removed. If neither i nor j become infectious during the outbreak or if both of these nodes become infectious at the exact same time, no decision about inclusion of the edge $\{i, j\}$ will be made. □

Sample solution for Exercise 6: (a) Will be given in our module *Mathematical models and theorems under the uniform mixing assumption.*

(b) The connected component of j^* is the subgraph induced by all nodes that experience infection. Its relative size is the final size of the simulated outbreak. $\square$